Department of Mathematics
Faculty of Science
Yarmouk University

Discrete Mathematics

Yarmouk University

Second Semester

2009/2010

Done by: Osama Alkhoun

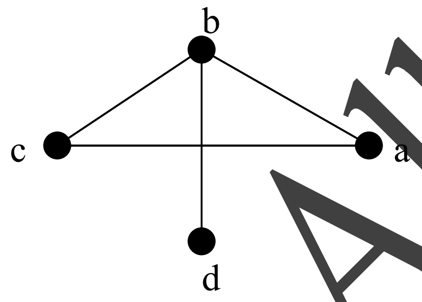
CHAPTER SIX:

FUNCTIONS.

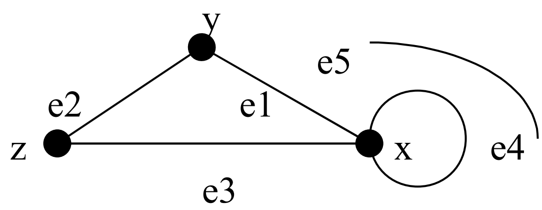
SECTION 6.1:
Basic Concepts.

Definition:
A graph G is a pair of finite sets { V, E } where V is called the set of nodes or vertices and E is called the set of edges with each edge e ∈ E we associate an ordered pair (a, b) they are called the endnodes of e.

Example:
Let V = { a, b, c, d } and E = { e1, e2, e3, e4 } where:
e1 = { a, b }
e2 = { b, c }
e3 = { b, d }
e4 = { a, c }
draw the diagram of graph { V, E }



Example:
Let G = { V, E } be a graph where:
V = { x, y, z }
E = { e1, e2, e3, e4, e5 }
e1 = { x, y }
e2 = { y, z }
e3 = { z, x }
e4 = { x, x }
e4 = { x, y }
draw the diagram of graph { V, E }



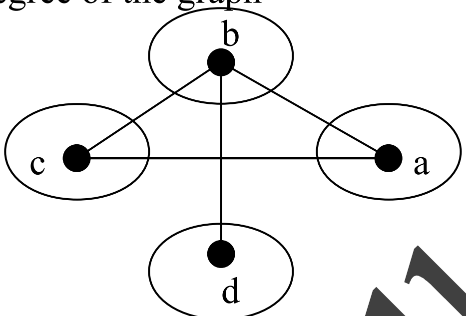
e4: is called self loop.
E1, e5: are called parallel edge.

Definition:
A graph with no self loop and no parallel edges is called a simple graph.

Definition:
Let $x \in V$ then degree (x) is the number of edges incident to x.

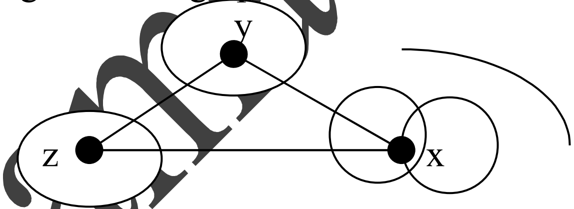
Theorem:
Let G = { V, E } be a graph, then $2|E|$ equal the sum of degree of the vertices.

Example:
The number of degree of the graph



the degree is:
$2 + 3 + 2 + 1 = 2|E|$
$8 = 2|E|$
$4 = |E|$ { e1, e2, e3, e4}

Example:
The number of degree of the graph



the degree is:
$5 + 3 + 2 = 2|E|$
$10 = 2|E|$
$5 = |E|$ { e1, e2, e3, e4, e5 }

Corollary:
In any graph, the number of edges with odd degree is even.
$2|E| = \deg(x1) + \deg(x2) + \ldots\ldots + \deg(xn)$

Example:
Find the number of vertices (nodes) in a graph with exactly 6 edges in which each node is of degree 2

$2|E|$ = the sum of degrees,

Let n be the number of nodes.

$2|E| = 2n$

$12 = 2n$

$n = 6$

Example:
Find the number of vertices (nodes) in a graph with exactly 12 edges, and 2 nodes degree 3, and the remaining of degree 2

$2|E|$ = the sum of degrees,

Let n be the number of vertices (nodes).

$2|E| = 2.3 + (n – 2).2$

$24 = 6 + 2n - 4$

$24 = 2n + 2$

$22 = 2n$

$11 = n$

Example:
Can we construct a graph with 12 degree and 2 nodes of degree 3, and 9 of degree 4.

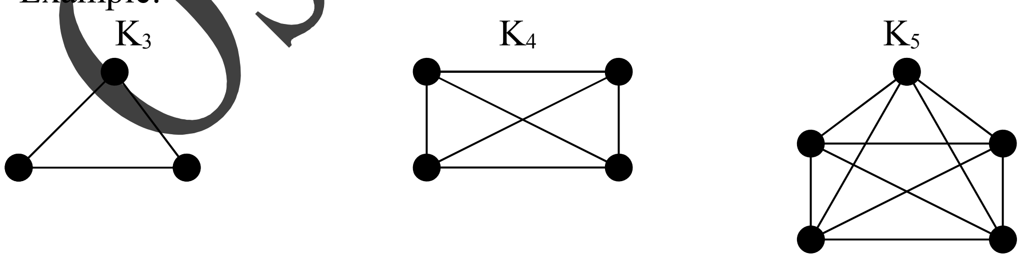
$2|E| = 2.3 + 9.4$

$24 = 6 + 36$

Then can't we construct a graph.

Definition:
A simple graph with n nodes in which every pair of distinct nodes is connected by an edges, is called complete graph of n nodes and denoted by K_m

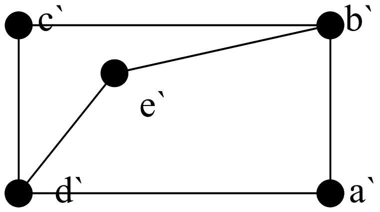
Example:

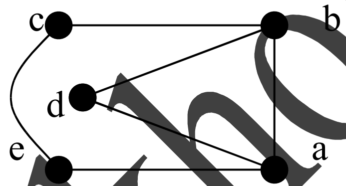


Note:
In K_n:

1. Degree (x) = n -1, for any node x
2. $|E| = \frac{n(n-1)}{2}$
3. $2|E| = n(n-1)$

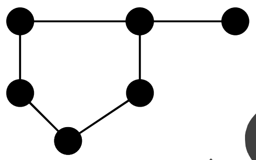
Example:
Find $|E|$ in graph with 5 nodes, and 2 of degree 3, and 3 of degree 2.
$2|E| = 2.3 + 3.2$
$2|E| = 6 + 6$
$2|E| = 12$
$|E| = 6$

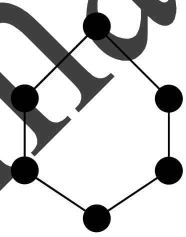




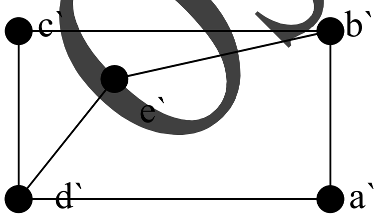
Definition:
Let G = { V, E }, G` = { V`, E` }, the graphs G and G` are called isomorphic if there exist bijection F_V: V → V` and $F_E$: E → E` such that if e is an edges in E with endnodes a and b, then $F_E(e)$ is an edge in G` with endnodes $F_V(a)$ and $F_V(b)$.

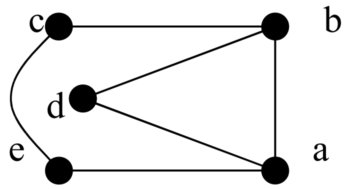
Example:





Not isomorphic





Not isomorphic

Example:

<u>F_V</u>

a → 1

b → 2

c → 3

d → 4

<u>F_E (edges).</u>

e1 → e1`

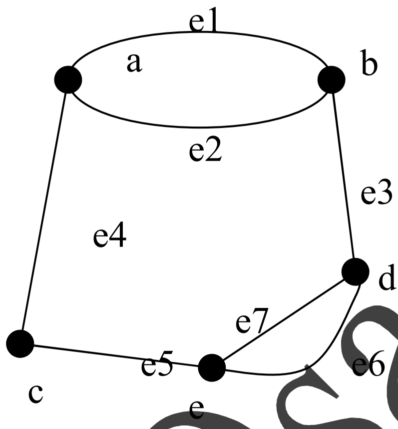
e2 → e2`

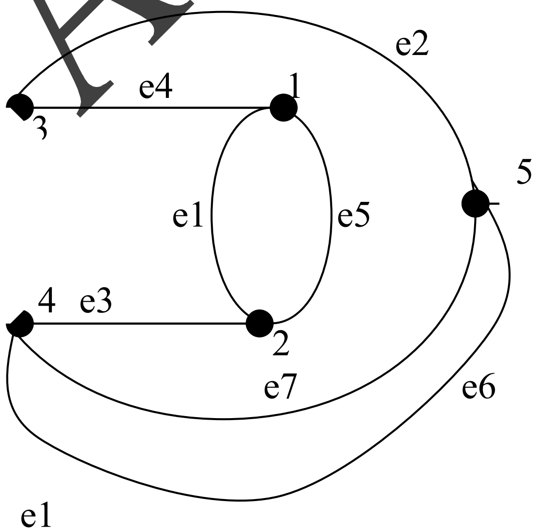
e3 → e5`

e4 → e3`

e5 → e4`

Example:





<u>F_V</u>

a → 1

b → 2

c → 3

d → 4

e → 5

<u>F_E (edges).</u>

e1 → e1

e2 → e5

e3 → e3

e4 → e4

e5 → e2

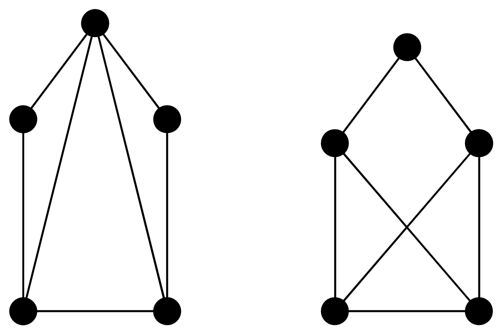
e6 → e6

e7 → e7

Propositions:

1. Suppose that G = { V, E } and G\` = { V\`, E\` } are isomorphic under the bijection $F_V$: V → V\` and F_E: E → E\`
   i. let x be a node in V, then the degree of x equal to the degree of F_V (x)
   ii. the graph G and G\` must possess the same number of nodes of any given degree
   iii. if e is an edge in G with endnodes a and b, then the endnodes of F_E(e) in G\` have the same degrees as a and b.

Example:



G5 G6

The graph shown cannot be isomorphic because G5 has a node of degree 4 whereas G6 does not.