Department of Mathematics
Faculty of Science
Yarmouk University

Discrete Mathematics

Yarmouk University

Second Semester

2009/2010

Done by: Osama Alkhoun

CHAPTER FIVE:

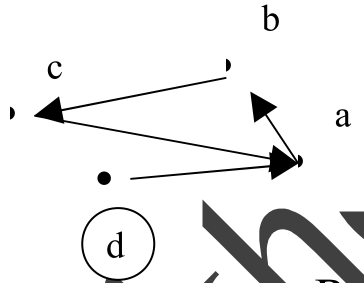
RELATIONS

SECTION 5.2:
Composition of Relations.

We defined the composition of relation (R) with itself written R . R by $x(R \circ R)y$ iff $\exists z \in A$ such that xRz and zRy

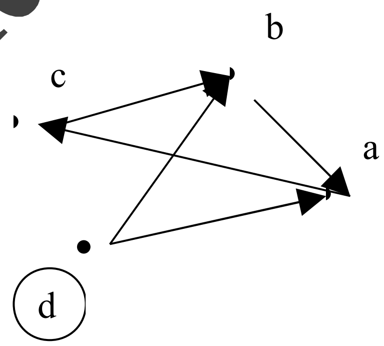
Example:
Let A = { a, b, c, d } and R = { (a, b) (b, c), (c, a), (d, d), (d, a) }
Find $R \circ R$?

Solution:
The diagraph of R is given by

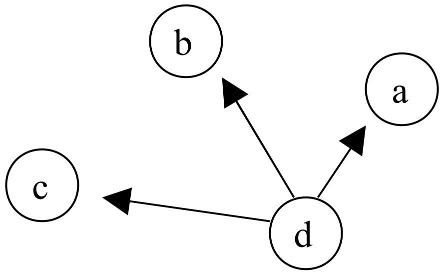


i. ${}_xR^0{}_y$ iff x = y
ii. ${}_xR^1{}_y$ iff $\exists$1 arrows from x to y = x(R)y = ${}_xR_y$
iii. ${}_xR^2{}_y$ iff $\exists$2 arrows from x to y = $x(R \circ R)y$
iv. ${}_xR^3{}_y$ iff $\exists$3 arrows from x to y = $x(R \circ R \circ R)y$
v. ${}_xR^4{}_y$ iff $\exists$4 arrows from x to y = $x(R \circ R \circ R \circ R)y$
vi. ${}_xR^5{}_y$ iff $\exists$5 arrows from x to y = $x(R \circ R \circ R \circ R \circ R)y$
vii. ${}_xR^n{}_y$ iff $\exists$n arrows from x to y = $x(R \circ R \circ R \circ R \circ \dots\dots \circ Rn)y$

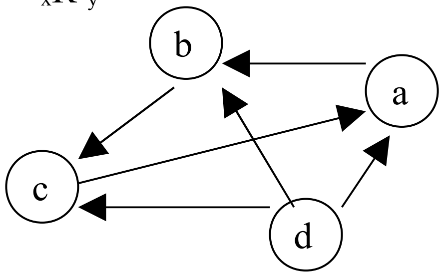
$R \circ R = x(R \circ R)y = {}_xR^2{}_y$



$R \circ R \circ R = x(R \circ R \circ R)y = {}_xR^3{}_y$



$R \circ R \circ R \circ R = x(R \circ R \circ R \circ R)y = {}_xR^4{}_y$



definition:

If R a relation on a set A, then $xR \circ Ry$ if $\exists z \in A$ such that ${}_xR_z$ and ${}_zR_y$ $xR \circ R \circ Ry = {}_x R \circ R^2{}_y$ if $\exists z \in A$ such that ${}_xR^2{}_y$ and ${}_zR_y$

In general:

We write ${}_xR^2{}_y$ if $\exists z \in A$ such that ${}_xR^{n-1}{}_y$ and ${}_zR_y$ we say that ${}_xR^0{}_y$ if $x = y$ and ${}_xR^1{}_y$ if ${}_xR_y$.

If M_R is the matrix represent the relation R1 then M_R X M_R is the matrix represents $R \circ R = R^2$

Example:

$$\text{Let } M_R = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

be the matrix represents a relation R

then M_R X M_R =

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

is the matrix represented $R \circ R$

NOTE:

The entry in the ith row and jth column of M_R X M_R is given by multiplying the ith row and jth column of M_R

In general:

The matrix of the relation R^n is given by $M^n{}_R$ where $M^n{}_R$ is the matrix M_R multiplied n – times