Department of Mathematics
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Discrete Mathematics

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CHAPTER FOUR

COUNTING
AND
COUNTABILITY.

SECTION 4.2:
Functions and Counting.

Proposition:
Suppose that A and B are two nonempty finite sets and $|A| = m$, $|B| = n$, there are n^m **function possible** from A to B

Example:
Let A = { a, b, c } B = { s, t }
There are 2^3 function from A to B and There are 3^2 function from B to A

- if $|A| > |B|$, there is no 1 – 1 function from A to B

  Example:
  Let A = { a, b, c } B = { s, t }
  How many injective from A to B ? = A > B → 0 (zero)
  How many injective from B to A ? = $3 \times 2 = 6$

- if $|A| = m$, $|B| = n$, $n \geq m$ **(**$|B| > |A|$**)**, then there are $\frac{n!}{(n-m)!}$ **injective (1-1)** function possible from A to B

  Example:
  Let A = { a, b, c } B = { s, t, v u, w }
  How many injective from B to A ? = 0 (zero)
  How many injective from A to B ? = $|A|$ = **m = 3**, $|B|$ = **n = 5**

$$= \frac{n!}{(n-m)!} = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

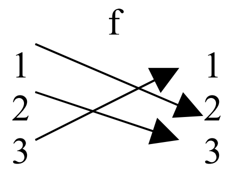
Corollary:
Suppose that A is a nonempty finite set with $|A| = n$, then there are n! different injective functions possible from A to A (from A to itself)

Proposition:
Suppose that $|A| > |B|$, and that f: A → B is a function then there are exist (s, t $\in$ A) with f (s) = f (t)

Definition:
Let A be a nonempty set a bijection f: A → A is called a **permutation**
Note:

f: A → A (if we can 1 – 1 then onto and via versa)

Example:
Let A = { 1, 2, 3 } and define f: A → A by



There are 3! A way for arrangement A

Remark:
If $|A| = n$, there are n! permutation.

Example:
In how many ways we can arrange the letters of word MATH?
There are 4! Way to arrange them
Note:
If there is repetition of the word, the division number of repetition for each element on both.

Example;
In how many ways we can arrange the letters of word MISTER?
There are 6! Ways to arrange the letter word mister.

Example;
In how many ways we can arrange the letters of word MISSISSIPPI?
There are $\frac{11\,!}{4!4!2!}$ Ways to arrange the letter word Mississippi.

Example;
In how many ways we can arrange the letters of word PASCAL?
There are $\frac{6!}{2!}$ Ways to arrange the letter word Pascal.

Example;
In how many ways we can arrange the letters of word LETTERS?
There are $\frac{7!}{2!2!}$ Ways to arrange the letter word Letters.

Example;
How many ways to arrangement a sequence of 6 identical white balls and 5 identical red balls.
There are $\frac{11!}{5!6!}$ Ways to arrange them.

Question 5 / page 145
In how many ways can we assign a number between 1 to 1oo to each of 50 people ?

1. if each people must be assigned a different number?
   with repeat number = 100^{50}

2. if no two numbers can have a difference of 1and each people must assigned a different number?
   the number = $\frac{100!}{(100-50)!}$