Department of Mathematics
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Discrete Mathematics

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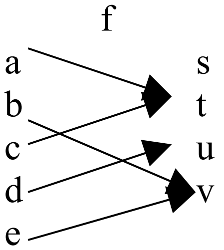
CHAPTER THREE:

FUNCTIONS.

SECTION 3.5:
Functions and Set Operations.

Definitions:
Let f: A → B be a function and let **S** be a subset of **A** and **T** be a subset of **B** ($S \subseteq A$, $T \subseteq B$), then f (S) = { y: y ∈ B and y = f (x): x ∈ S } is called **the direct image of S under f** and f^{-1} (T) = { x: f (x) ∈ T } is called **the inverse image of T under f**

Example:
Let A = { a, b, c, d, e }   B = { s, t, u, v } and
Let f: A → B be defined by



let S = { a, c, d } ⊆ A   T = { s, t, v } ⊆ B

- f (S) = { f (x) : x ∈ S }
  = { f (a), f (c), f (d) }
  = { t , u }

- f^{-1} (T) = { x ∈ A: f (x) ∈ T }
  = { a, c, b, e }
  D = { a, b } → f (D) = { t, v }
  C = { s, v } → f^{-1} (C) = { b, e }

Example:
Let f: R → R is given by f (x) = 2x + 1
Let S = { x: $-2 \le x \le 2$ }   T = { x: $0 \le x \le 2$ }
Find f (S), f^{-1} (T)
Solution:
f (S) = { f (x): x ∈ S }
= { f (x): $-2 \le x \le 2$ }
then = { f (x): $-3 \le x \le 5$ }
f^{-1} (T) = { x: f (x) ∈ T }
= { x: $0 \le f(x) \le 2$ }
= { x: $0 \le 2x + 1 \le 2$ }
= { x: $-1 \le 2x \le 1$ }
= { x: $-\frac{1}{2} \le x \le \frac{1}{2}$ }

Example:
Let f: R → R is given by f (x) = x^2
Let A = { x: $-1 \le x \le 3$ }
Find:

i. f (A)
ii. f^{-1} (A)
iii. f (R)
iv. f^{-1} (R)
v. f^{-1} (f(A))
vi. f ({ 4 })

Solution:

i. f (A) = { f (x) : x ∈ A }
= { f (x) : $-1 \le x \le 3$ }
= { x: $0 \le x \le 9$ }

ii. f^{-1} (A) = { x ∈ R: f (x) ∈ A }
= { x ∈ R: $-1 \le f(x) \le 3$ }
= { x ∈ R: $-1 \le x^2 \le 3$ }
= { x ∈ R: $-1 \le |x| \le \sqrt{3}$ }
= { x ∈ R: $-\sqrt{3} \le |x| \le \sqrt{3}$ }

iii. f (R) = { f (x) : x ∈ R }
= { x^2 : x ∈ R }
= { [0, ∞) = R }

iv. f^{-1} (R) = { x ∈ R: f (x) ∈ R }

v. f^{-1} (f(A)) = f^{-1} ({x: $0 \le x \le 9$ })
= { x: $-3 \le x \le 3$ } ≠ A

vi. f ({ 4 }) = { 16 }

vii. f^{-1} ({ 4 }) = { -2, 2 }

Remark:
If f: A → B is a function, then f(A) = B iff f is surjective.

Example:
Let f: R → R be defined by f (x) = $\frac{2x-1}{3}$, find f^{-1} if possible.

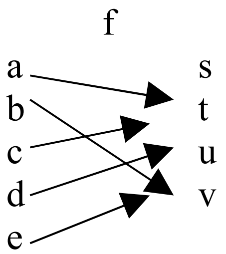
A = { x ∈ R: $0 \le x \le 2$ } B = { x ∈ R: $1 \le x \le 3$ }

f^{-1} (A) = { x ∈ R: $\frac{-1}{3} \le x \le 1$ } f^{-1} (B) = { x ∈ R: $\frac{1}{3} \le x \le \frac{5}{3}$ }

Theorem:
Suppose that f: A → B is a function and H, K two sets of A, then
$f(H \cup K) = f(H) \cup f(K)$

Proof:
Let y be an element of $f(H \cup K)$
We can find an element x in $(H \cup K)$ such that $f(x) = y$ because $x \in H$ or K
And $y \in f(H)$ or $f(K)$, and we have shown that $f(H \cup K) \subset f(H) \cup f(K)$

Example:
Let A = { a, b, c, d, e } B = { s, t, u, v } and define f: A → B by



let H = { a} K = { b, d } D = { c }

- $f(H) = \{ t \}$
- $f(K) = \{ v, u \}$
- $H \cup K = \{ a, b, d \}$
- $f(H \cup K) = \{ t, v, u \}$
- $D^c = \{ a, b, d, e \}$
- $f(D^c) = \{ t, v, u \}$
- $f(D) = \{ t \}$
- $f(H) = \{ t \}$
- $(f(D))^c = (\{ t \})^c = \{ s, u, v \}$

- $H \cap D = \Phi$
- $F(H \cap D) = \Phi$
- $F(H) \cap f(D) = \{ t \}$

NOTE:
In general, it is **not true** that:
1. $f(S \cap T) = f(S) \cap f(T)$
2. $f(S^c) = (f(S))^c$

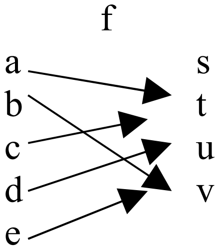
then:
- $f(S \cap T) \neq f(S) \cap f(T)$
- $f(S^c) \neq (f(S))^c$

then the distribution be to the Union (∪), and **not true** to the Intersection (∩) and **complement**.

Theorem:
Suppose that f: A → B is a function and S, T two subsets of B, then

1. $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
2. $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$
3. $f^{-1}(S^c) = (f^{-1}(S))^c$

Example:
Let A = { a, b, c, d, e }   B = { s, t, u, v } and define f: A → B by



let S = { v}   T = { t, u }
Find:

1. $f^{-1}(S \cup T)$
2. $f^{-1}(S \cap T)$
3. $f^{-1}(S^c)$

1. $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
   $= \{ b \} \cup \{ a, d \}$
   $= \{ a, b, d \}$

2. $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$
   $= \{ b \} \cap \{ a, d \}$
   $= \Phi$

3. $f^{-1}(S^c) = (f^{-1}(S))^c$
   $= (\{ b \})^c$
   $= \{ a, c, d, e \}$

key concepts

SECTION 3.1: Functions.

1. function
2. $f: A \rightarrow B$
3. A : Domain
4. B : CoDomain
5. function value $y = f(x)$ for $x \in A$ and $y \in B$
6. identity function id_A
7. inductively defined function
8. recurrence relation
9. recursively defined function

SECTION 3.2: Injective and surjective functions.

1. injective function (one – to – one)
2. surjective function (onto)
3. bijective function (one-to-one correspondence)

SECTION 3.3: Composition of functions.

1. composition $g \circ f$
2. f^n, f composed with itself n times

SECTION 3.4: Inverse functions.

1. f^{-1}, the inverse of f
   a. $f^{-1} \circ f = id_A$
   b. $f \circ f^{-1} = id_B$

SECTION 3.5: Functions and Set Operations.

1. $f(S)$, the direct image of the set $S \subset A$: $f(S) \subset B$
2. $f^{-1}(Q)$, the inverse image of the set $Q \subset B$: $f(Q) \subset A$