Department of Mathematics
Faculty of Science
Yarmouk University

Discrete Mathematics

Yarmouk University

Second Semester

2009/2010

Done by: Osama Alkhoun

CHAPTER THREE:

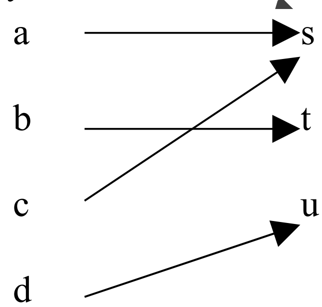
FUNCTIONS.

SECTION 3.1:
Functions.

Definition:
Let A and B be two sets, **A function** f from A to B (f: A → B) is rule that assign to each element x in A exactly one element y in B, we call **A** the **Domain** of (f) and **B** the **CoDomain** of (f).

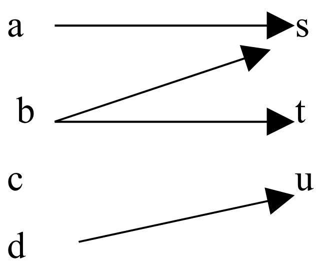
We write:
f: A → B, if x is an element of A and y is an element of B assigned to x, we write y = f (x) and call y the function value of f at x.

Example:
A = { a, b, c, d }
B = { s, t, u }
Let f : A → B be defined by:
f (a) = s
f (b) = t
f (c) = s
f (d) = u



Note:
1. A is a Domain, and B is a CoDomain
2. Is a function with Dom(f) = A, CoDom(f) = B

is not a function.
f (c) = ???
???: No CoDomain
And f (b) = s, f (b) = t
is more one CoDom



Define:
g : A → B by g (a) = t, g (b) = s, g (c) = u, g (d) = t this function
Dom (g) = A, CoDom (g) = B

Define:
Number of functions can be defined from A to B

$$|CoDom|^{|Dom|} = |B|^{|A|}$$

Example:
How many functions can be defined from A to B ?
h : A → B ?.
The cardinality of A = 4
The cardinality of B = 3

$$|CoDom|^{|Dom|} = |B|^{|A|} = |3|^{|4|} = 3 \times 3 \times 3 \times 3 = 81$$

h : A → B = 81 functions.

Example:
Let A = { 1, 2 }
How many functions can be defined with Dom (A) and CoDom (A) ?

$$|CoDom|^{|Dom|} = |A|^{|A|} = |2|^{|2|} = 2 \times 2 = 4$$

NOTE:
f = g, iff Dom (f) = Dom (g), CoDom (f) = CoDom (g) and f (x) = g (x)

Question 1:
determine which the following rules define the function ?

a.

Is function	Is not function f(1) = ???? is more one CoDom ????: is NO CoDom	Is function	Is not function f(1) = 1 f(1) = 2 is more one CoDom

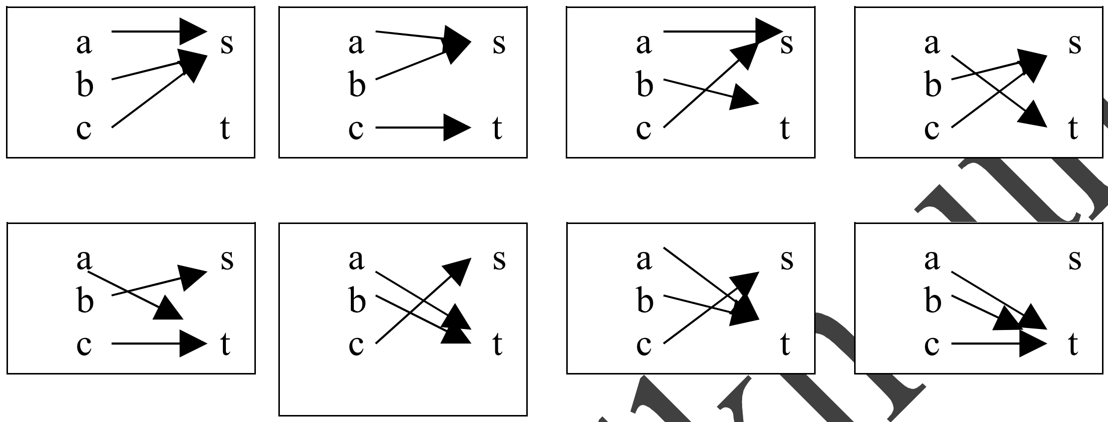
b.
f : R → Z, where f (x) is the integer nearest x.
f (2.5) = 2, 3 is not function. (have two CoDom)

c.
f : R → R, where f (x) is any real number y such that $y^3 = x$.
then is function. (always have One CoDom)

Question 2:
Let A = { a, b, c }
B = { s, t }
List all functions from A and B, and from B and A.

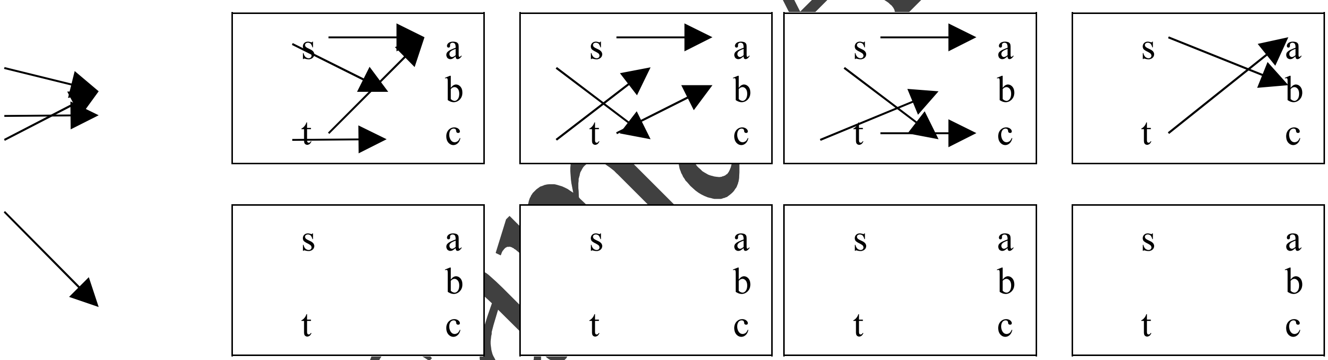
1. A and B (f: A → B)

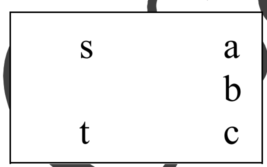
$$|CoDom|^{|Dom|} = |B|^{|A|} = |2|^{|3|} = 2 \times 2 \times 2 = 8$$



2. B and A (f: B → A)

$$|CoDom|^{|Dom|} = |A|^{|B|} = |3|^{|2|} = 3 \times 3 = 9$$





Example:
Let f : R^+ → R defined by $f(x) = \dfrac{x - 1}{x + 1}$

f (2) = 1/3
f (0) = -1
f (-2) = not defined -2 ∉ R^+

Example:
Let f: R → R defined by
f (x) = x^2 + 2 if x > 1
f (x) = 3x - 2 if x ≤ 1

solution:

f (0) where $0 \leq 1$
f (0) = $3 \times 0 - 2$ if $0 \leq 1$
= 0 - 2
= -2

f (4) where 4 > 1
f (4) = $4^2 + 2$ if 4 > 1
= 16 + 2
= 18

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x > 1 \\ 3x - 2 & \text{if } x \leq 1 \end{cases}$$

$$f(a+1) = \begin{cases} (a+1)^2 + 2 & \text{if } (a+1) > 1 \\ 3(a+1) - 2 & \text{if } (a+1) \leq 1 \end{cases}$$

$$f(a+1) = \begin{cases} (a+1)^2 + 2 & \text{if } a > 0 \\ 3(a+1) - 2 & \text{if } a \leq 0 \end{cases}$$ subtraction 1 from (a+1)

$$f(a+1) = \begin{cases} a^2 + 2a + 3 & \text{if } a > 0 \\ 3a + 5 & \text{if } a \leq 0 \end{cases}$$

Question 3:
Let A = { x : $x \geq 1$ }

f : A → R^+, be defined by f (x) = $\frac{x^2 + 1}{x}$
Find f (2), f (1.5), f (-1), if possible, for what values of z is f (z + 2) defined

solution:
f (2) = $\frac{x^2 + 1}{x} = \frac{2 \times 2 + 1}{2} = \frac{5}{2}$
f (1.5) = $\frac{x^2 + 1}{x} = \frac{1.5 \times 2 + 1}{2} = \frac{3.25}{2}$

f (-1) = not defined (must **$x \geq 1$** from defined)
f (z + 2) $\in$ A, iff z + 2 $\geq$ 1
z $\geq$ -1

Example:
let f: R → Z by f (x) is the greatest integer less than or equal x (f (x) = [x])

1. f (0.5)
   f (0.5) = [0.5]
   = 0
2. f (1.2)
   f (1.2) = [1.2]
   = 1
3. f (-0.3)
   f ((-0.3) = [(-0.3]
   = -1

1. for what values of x is f (x) = 2 ? [2, 3)
2. for what values of x is f (x) = -3 ? [-3, -2)

Definition:
Let A be any set and define f : A → A by f (x) = x for each x $\in$ A, then function f is called the **identity function** on A denoted by **id_A**

Example:
Let f: N → N defined by f (n) = $\forall$n $\in$ N then f is **identity function** in N, **id_N**.