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Discrete Mathematics

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Chapter Two

SET THEORY

Section 2.6

Cartesian product

Definition:

Let A and B be sets, the Cartesian product of A and B, denoted by $A \times B$, is the set of all ordered pairs (x, y) , where x is an element of A and y is an element of B
That is $A \times B = \{ (x, y) : x \in A \text{ and } y \in B \}$

$$A = \{ a, b \}$$

$$B = \{ 1, 2 \}$$

The cardinality of A = 2

The cardinality of B = 2

Then the number of pair elements = $2 * 2 = 4$

$$A \times B = \{ (a, 1), (a, 2), (b, 1), (b, 2), \}$$

Cardinality:

If $|A| = n$, $|B| = m$, then $|A \times B| = n \times m$

Example:

$$A = \{ 1, 2, 3 \}$$

The cardinality of A = 3

The cardinality of A = 3

Then the number of pair elements = $3 * 3 = 9$

$$A \times A = \{ (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3) \}$$

Applying in Cartesian product:

$$A \times B \neq B \times A$$

Example:

$$\text{Let } A = \{ 1, 2, 3 \}$$

$$A \times A = \{ (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3) \}$$

Example:

$$A = \{ a, b \}$$

$$B = \mathbb{Z}$$

$$A \times B = \{ \dots, (a, -1), (a, 0), (b, -1), (b, 0), \dots \}$$

Example:

$$A = \mathbb{R}$$

$$B = \{ 1 \}$$

$$A \times B = \{ (x, y) : x \in A \text{ and } y \in B \}$$

$$\mathbb{R} \times \{ 1 \} = \{ (x, y) : x \in \mathbb{R} \text{ and } y \in \{ 1 \} \}$$

$$= \{ (x, 1) : x \in \mathbb{R} \}$$

Example:

$A = \phi$ (empty set)

$B = \mathbb{N}$

The cardinality of $A = 0$

The cardinality of $B = (0, \infty)$

Then the number of pair elements $= 0 * (0, \infty) = 0$

$|A \times B| = 0$, nothing element ($= 0$) (zero element)

Example:

Let $D = \{ (1,1), (2,1) \}$

Then $A = \{ 1, 2 \}$

$B = \{ 1 \}$

$A \times B = \{ (x, y) : x \in A \text{ and } y \in B \}$
 $= \{ (1,1), (2,1) \}$

Example:

Let $D = \{ (1,1), (2,2) \}$

Then $A = \{ 1, 2 \}$

$B = \{ 1, 2 \}$

$D \neq A \times B$ for any A and B

$A \times B \times C = \{ (x, y, z) : x \in A, y \in B \text{ and } z \in C \}$

$A = \{ 1, 2 \}$

$B = \{ 3 \}$

$C = \{ 4, 5 \}$

The cardinality of $A = 2$

The cardinality of $B = 1$

The cardinality of $C = 2$

Then the number of pair elements $= 1 * 2 * 2 = 4$ element

$A \times B \times C = \{ (1, 3, 4), (1, 3, 5), (2, 3, 4), (2, 3, 5) \}$

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