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Discrete Mathematics

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Chapter Two

SET THEORY

Section 2.5

Power Sets.

A = { a, b }
Is A1 = { a } subset of A ? YES
Is A2 = { b } subset of A ? YES
Is A3 = { } subset of A ? YES
Is A4 = { a, b } subset of A ? YES

NOTE:
Number of subset for any set is 2 exponentiation number of set (2^n)

P (A) = { ϕ, A , {a}, {b} } → (2^2) = 4
OR P (A) = { {} , { a, b }, {a}, {b} } → (2^2) = 4

Definition:
Let A be any set, the power set of A, denoted by P (A) is the set of all subsets A.

Example:
Let A = { 1, 2, 3 }
The power of set is → (2^3) = 8
P (A) = { ϕ, A , {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3} }
The cardinality (absolute):

$|P(A)| = 8$

$|A|$ = the number of elements of A

If $|A|$ = n (elements) , then $|P(A)| = (2^n)$

Example:
Let B = ϕ   ϕ is subset of B
P (B) = { ϕ } = { B } The power of set is → (2^0) = 1 element

Example:
A = { a, b, c, d, e }

$|P(A)| = (2^5) = 32$

a ∈ A   a ∉ P (A)
{a} ∉ A   {a} ∈ P (A)

If a Subset S ⊂ A → then the subset S ∈ P (A)
The **elements** {b, d} ⊂ A → the **element** of {b, d} ∈ P (A)
{b, d} is **group of elements** that exist in A set.
{b, d} is **one element** that exist in P (A) set.

→ B = { b, c, d }
B ⊂ A → B ∈ P (A)
B ∉ A

Example:
Let A = { {x}, {y, w}}
{x} ∈ A
{y, w} ∈ A
{{x}} ⊆ A
{{y, w}} ⊆ A
P (A) = { ϕ , {{x}}, {{y, w}}, {{x},{y, w}}}

Question 3 / page 69 in Book:
A = { a, b, c, d, e }
B = { a, b }
C = { B, ϕ }
D = { a, b, {a, b }}

A ∩ B = B = { a, b } → A ⊂ B **then** A ∩ B = B
C ∩ D = {{a, b }} = { B }
A ∩ D = { a, b }
C ∩ P (A) = { B, ϕ } = C
D ∩ P (A) = {{a, b }} = { B }

→ Indicate whether each of the following is true or false
A = { a, b, c, d, e }
B = { a, b }
C = { B, ϕ }
D = { a, b , {a, b }}

1. A ∈ P (A) → true (X ∈ P (Y) if X ∈ Y)
2. C ⊂ P (A) → true
3. D ⊂ P (A) → false
4. B ⊂ D → true
5. B ∈ D → true
6. { a, b } ∈ C → true

Example:

$A = \{ a, b, 2, 3 \}$

The power set is $2^4 = 16$ elements.

$P(A) = \Big\{ A, \phi, \{a\}, \{b\}, \{2\}, \{3\}, \{a, b\}, \{a, 2\}, \{a, 3\}, \{b, 2\}, \{b, 3\},$
$\{2, 3\}, \{a, b, 2\}, \{a, 2, 3\}, \{a, b, 3\}, \{b, 2, 3\} \Big\}$

$C = \Big\{ \{a\}, \{b\}, \{c\} \Big\}$

The power set is $2^3 = 8$ elements.

$P(C) = \Big\{ C, \phi, \big\{\{a\}\big\}, \big\{\{b\}\big\}, \big\{\{c\}\big\}, \big\{\{a\},\{b\}\big\}, \big\{\{a\},\{c\}\big\}, \big\{\{b\},\{c\}\big\} \Big\}$

$D = \Big\{ \phi, \{ \phi \} \Big\}$

The power set is $2^2 = 4$ elements.

$P(D) = \Big\{ D, \phi, \{ \phi \}, \big\{\{ \phi \}\big\} \Big\}$

Question 4 / page 69 in Book

Prove that if $A \subset B$, then

$$P(A) \subseteq P(B)$$

$S \in P(A)$

want: $S \in P(B)$????

$A \subset B$: all elements in $A \in B$

Suppose x element in A

$x \in A$ and $x \in B$

$P(A) = \Big\{ \{x\}: x \text{ is an element in } A \Big\}$

$\therefore P(B) = \Big\{ \{x\}: x \text{ is an element in } A \Big\} = \Big\{ P(A) \Big\}$

$$\therefore P(A) \subseteq P(B)$$