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Discrete Mathematics

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Chapter Two

SET THEORY

Section 2.3
Set Operations

Suppose B = { x : x is **not** a natural number }

Natural number: 1, 2, 3, 4, ……

Is $0 \in B$? yes
Is $1 \in B$? no
Is $1/2 \in B$? yes
Is $-1 \in B$? yes
Is the letter 'a' $\in B$? yes

- In order the B is well defined, we must stipulate the universal set U. of which all subsets under discussion are considered to be subsets.
- If B is defined in set Builder – Notation then the universal set is the universe of discourse of the predicate.

Example:
S = { x : P (x) is true }
U = Z
$B = \{ 0, -1, -2, -3, -4, \ldots \} \subset Z$

U: Universal set
U = R
$B = (-\infty, 1) \cup (1,2) \cup (2,3) \cup (3,4) \cup \ldots\ldots\ldots$

Definition:
Let A be a subset of universal set U
The set { x: x is in U and $x \notin A$ } is called the **complement** of A and denoted be A^c

Example:
Let U = R
$A = \{ x : 1 \le x \le 2 \}$
$A^c = \{ x : x < 1 \text{ and } x > 2 \}$
$= (-\infty, 1) \cup (2, \infty)$

Example:
Let U = Z
A = { x : x is rational }
A = Z (All numbers)
$A^c = \varphi$ (empty set)
$B = \varphi$ (empty set)
$B^c = Z$ (All numbers)

Note:
The empty set is contained in every other set.
A is empty set, and B contain any elements then ($A \subset B$)

Example:
$D = \{ x : x < 1 \text{ and } x > 2 \}$
$D = \varphi$

Definition:
Let A, B be sets
The **UNION** of A and B ($A \cup B$) in the set $\{ x : x \in A \text{ or } x \in B \}$
The **INTERSECTION** of A and B ($A \cap B$) is the set $\{x : x \in A \text{ and } x \in B \}$

Example:
$A = \{ a, b, c, d \}$
$B = \{ b, c, e, f \}$

$(A \cup B) = \{ x : x \in A \text{ or } x \in B \}$
$= \{ a, b, c, d, e, f \}$
$(A \cap B) = \{ x : x \in A \text{ and } x \in B \}$
$= \{ b, c \}$

Example:
U = R (universal set)
$A = \{ x : 0 < x < 2 \}$
$B = \{ x : 1 < x < 3 \}$
$C = \{ x : 2 < x < 4 \}$

$(A \cup B) = \{ x : 0 < x < 3 \}$
$(A \cap B) = \{ x : 1 < x < 2 \}$
$(A \cap C) = \{ x : \{ \} \}$ **OR** $= \{ x : \varphi \}$
$(B \cap C) = \{ x : 2 < x < 3 \}$

Definition:
Two sets A and B are called **disjoint** if $A \cap B = \varphi$
Then:
Let A be a subset of U, then:

1. $A \cap A^c = \varphi$
2. $A \cup A^c = U$

Example:
Let A and B be sets of Integers defined by :
A = { x : x is divisible by 2 }
B = { x : x is divisible by 3 }

$(A \cup B) = \{ x : x \text{ is divisible by 2 or 3} \}$
$\{ 2, 3, -2, -3, 6, -6, 12, -12 \ldots\ldots. \}$
$(A \cap B) = \{ x : x \text{ is divisible by 2 and 3} \}$
$\{ 6, -6, 12, -12, 18, -18 \ldots\ldots. \}$

$A^c = \{ x : x \text{ is not divisible by 2} \}$

$B^c = \{ x : x \text{ is not divisible by 3} \}$

$(A \cup B)^c = \{ x : x$ not divisible by 2 **and** not divisible by $3 \}$

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

Theorem 1:
Distributive Laws

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Theorem 2:
DeMorgan's laws

1. $(A \cup B)^c = A^c \cap B^c$
2. $(A \cap B)^c = A^c \cup B^c$

Definitions:
Let S and T be sets. The set difference at S and T, denoted by S\T, is the set
$(S \cap T^c) = \{ x : x \in S \text{ and } x \notin T \}$

Example:
A = { 1, 2, 3, 4 }
B = { 3, 4, 5, 6 }
A\B = { 1, 2 }
B\A = { 5, 6 }

Example:
IS the S, T two sets
S\T and T\S are **disjoint**
→ $S \setminus T \cap T \setminus S = \varphi$

Example:
$A = \{ 1, 2 \}$
$B = \{ 1, 2, 3 \}$
$A \setminus B = \varphi$ If $A \subset B$, then $\mathbf{A \setminus B = \varphi}$
Remark:
$(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

Theorem:
$(A \cup B) = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$
$(A \cup B)$: written are **UNION** of disjoint sets.
Important Identities:
idempotent laws
1. $A \cup A = A$
   $A \cap A = A$
2. $A \cup \varphi = A$
   $A \cap \varphi = \varphi$
3. $A \cup U = U$
   $A \cap U = A$
4. $A \cup A^c = U$
   $A \cap A^c = \varphi$

Associativity
5. $(A \cup B) \cup C = A \cup (B \cup C)$
   $(A \cap B) \cap C = A \cap (B \cap C)$

Commutativity
6. $A \cup B = B \cup A$
   $A \cap B = B \cap A$

Distributive laws
7. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
8. $(A \cup B)^c = A^c \cap B^c$
   $(A \cap B)^c = A^c \cup B^c$
9. $(A^c)^c = A$
10. $(A \cup B) = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$
11. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
12. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$