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Discrete Mathematics

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Chapter one

LOGIC

Section 1.6
Proof in mathematics

1. Direct Proof

P_1 , P_2 , P_3 , ……., P_n → C: is rational numbers

Theorem 1: if x is even integer, then x^2 is also even.

PROOF:

Suppose that x is even integer

x = 2n: for some integers

$x^2 = 4n^2 = 2(2n^2)$ therefore x^2 is also even.

x^2 is even

2. Contrapositive

Theorem 2: if x is integer and x^2 is even, then x is even.

: x^2 even ⇒ x is even.

PROOF:

Suppose that x is odd integer

$x = 2n + 1$

$x^2 = (2n + 1)^2$

$x^2 = 4n^2 + 4n + 1$

∴ x^2 is odd.

Theorem 3: If <u>x^2 is even</u>, then <u>x is even</u>.

Contrapositive Direct Proof

3. Biconditional

$P \Leftrightarrow Q$ equivalent $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

Theorem 4:

Let x be an integer, then <u>x is even</u> **<u>if and only if</u>** <u>x^2 is even</u>

$P \Longleftarrow Q$

(⇒) suppose x is even (**Theorem 1**: direct Proof)

(⇐) x^2 even, then x is even (**Theorem 2**: Contrapositive)

4. The Contradiction

$P \Rightarrow Q$ → $P \wedge \sim Q$ (negation)

$P_1 \wedge P_2 \wedge P_3 \wedge \ldots\ldots \wedge P_n \Rightarrow C$ → $P_1 \wedge P_2 \wedge P_3 \wedge \ldots\ldots \wedge P_n \wedge \sim C$

Theorem 5:

There is no rational number x such that $x^2 = 2$

If <u>x rational</u>, then <u>$x^2 \neq 2$</u>

P Q negation wanted ($P \wedge \sim Q$)

PROOF:
Suppose x is rational and $x^2 = 2$

$x = \frac{a}{b}$ With a and b have no common factor

$$x = \frac{a}{b}$$

$$x^2 = \frac{a^2}{b^2}$$

$$2 = \frac{a^2}{b^2}$$

$a^2 = 2\,b^2$
theorem :
a^2 is even → a is even
a = 2 n
→ $a^2 = 4\,n^2$
From $a^2 = 2\,b^2$
$4\,n^2 = 2\,b^2$
$2\,n^2 = b^2$
b^2 is even → b is even
a and b have 2 as common factor.

5. Counter Examples
Example:
$\forall x\, P(x)$ is false.

- All multiples of 3 are odd **false**

counter example :
6 is multiple of three which is even

- All odd numbers are divisible by 3. **False**

counter example:
7 is not divisible by 3.

- If P is an odd prime, then P + 2 is also prime **false**

counter example:
7 is prime odd but 7 + 2 = 9 is not prime.

- The square root of any integer is irrational **false**

counter example :
$\sqrt{4} = 2$ rational.

Key concepts:

CHAPTER ONE:

LOGIC.

SECTION 1.1:

Propositions.

- Propositions
- Truth value
- Logical variable
- Logical connectives: $\wedge$ ("and"), $\vee$ ("or")
- Negation ~
- Propositional form
- Truth table
- Logical equivalence
- Logical identities: DeMorgan's Laws and the distributives laws

SECTION 1.2:

The conditional and Biconditional.

- Conditional connectives: $\Rightarrow$ (implication)
- Converse and contrapositive
- Biconditional: $\Leftrightarrow$ (if and only if)
- Tautology, contradiction, and contingency.

SECTION 1.4:

Predicates.

- Predicate
- Universe of discourse
- Binding
- Quantification: universal ($\forall x$) and existential ($\exists x$)

SECTION 1.6

Proof in Mathematics.

- Direct proof
- Contrapositive
- Biconditional
- Contradiction
- Counter example