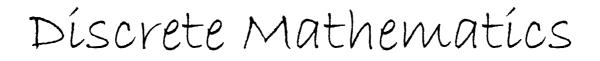
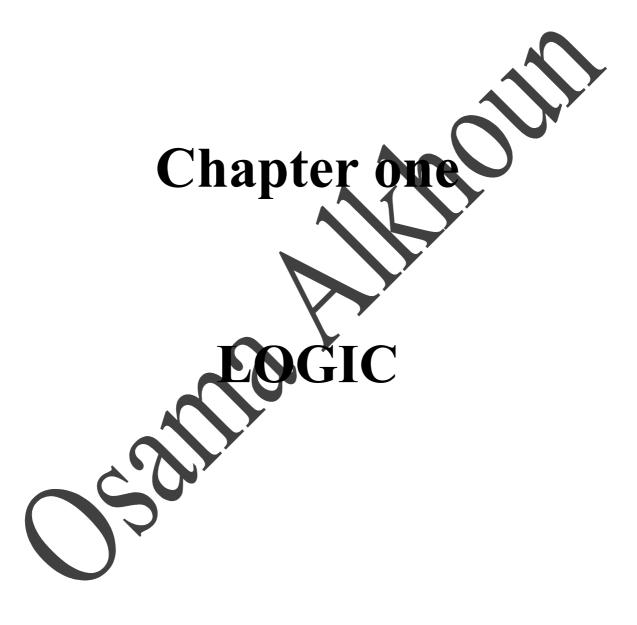
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Section 1.4 **Predicate.**

X is even integer : not proposition. It becomes a proposition only when the value of has been determined. Set x=4, then we have a true proposition Set x=5, then we have a false proposition

An assertion that contains one or more variables is a predicate P(x), P(x,y): is a predicate. P(2): is a proposition. P(5): is a proposition.

The set that we choose the values of x from it is called the **University of Discourse**.

Example:

Let the **Universe of discourse** be the set of integers, and let P(x,y) be the predicate "x.y = 12"

P (1,3): "1.3=12": "3=12" false proposition P (2,6): "2.6=12": "12=12" true proposition

Example:

Let Q(x) be the predicate "2.x is even integer" and the **universe of discourse** be the set of integers.

Q(x) is true for all x in the universe of discourse

 $\forall x Q(x)$: for all x in the universe of discourse Q(x) hold.

 $\forall x Q(x)$: is a proposition.

Example:

Q(x): 2.x is even integers, universe of discourse all integers.

Q (1), Q (2), Q (9), Q (11) : is true.

NOTE: that for all values of \mathbf{x} , Q (x) is true.

Clarification:

A: "for all values of x, Q (x) is true OR hold"

A: is true proposition.

In general: "for all values of x, P (x) is true" is proposition.

We denote for all by \checkmark

A: $\forall x Q(x)$

A: for all values of x, Q (x) is true

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Example:

Let Q(x) be the predicate "x.1= x", and universe of discourse be all real numbers.

 $\forall x Q(x)$: "for all values of x, x.1=x". is true proposition

Example:

Let P(x) be the predicate "x.1=0", and universe of discourse be all real numbers.

 $\forall x \ Q(x)$: "for all values of x, x.1=0". is false proposition But there is a value of x such that P (x) hold.

We denote There exists : By \exists

 $\exists x P(x)$:"there exists a value of x such that P(x) hold is true proposition

Example:

Let the universe of discourse be the real numbers.

 $\begin{array}{l} P(x): "\; x^2 \geq 0 \; " \\ Q(x): "\; 3.x > 10" \end{array}$

1. $\forall x P(x)$: "for all values of x, $x^2 \ge 0$ "True (1)2. $\forall x Q(x)$: "for all values of x, 3 x > 10"False (0)3. $\exists x P(x)$: "there exists a value of x such that $x^2 \ge 0$ "True (1)4. $\exists x Q(x)$: "there exists a value of x such that 3.x >True (1)10"True (1)

Example: Let P (x, y) be the predicate "x > y" and let the universe of discourse be the set of real numbers.

- 1. If $\forall y P(x,y)$: "there exists a value of x such that for all values of y, $x > y \rightarrow$ is false (0).
- 2. $\exists x \exists y P(x,y)$: "there exists a value of x and a values of y,

 $x > y \rightarrow is true (1).$

3. $\forall x \exists y P(x,y)$: "for all values of x, there exists a value of y, Such that $x > y \rightarrow$ is true (1).

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Example:

Let the universe of discourse be the set integers. a. $\forall x \exists y [x + y = 0]$: "for all x, there exists a value of y such that x + y = 0 " \rightarrow is true (1). b. $\exists x \forall y [x,y=0]$: "there exists value of x, such that for all of y, $x.y = 0 " \rightarrow is true (1)$ c. $\exists x \exists y [x,y=15]$: "there exists value of x, and a value of y such that $x.y = 15" \rightarrow is true (1).$ d. $\forall x \forall y [x, y > 0]$: "for all x and for all y, x, y > 0" \rightarrow is false (0). Example: P(x,y): "x.y = 4 " $\forall x \exists y P(x,y)$: "for all x, there exists a value of y such that \rightarrow is false (0). NOW: If P(x) is a predicate then the negation of $\mathbf{P}(\mathbf{x})$ is gives by and the following : $\sim [\forall x P(x)] = \exists x (\sim P(x))$ $\sim [\exists x P(x)] = \forall x (\sim P(x))$ Example: Let Q (x) be the predicate " $x^2 \ge 0$ " and let the universe of discourse be the set of integers. $\forall x Q(x)$: is false (0). ~ ($\forall x Q(x)$): is true ($\exists x [x^2 \le 0])$ That's mean: $\exists x (\sim \mathbf{O}(x))$ mbers ral number (N): Integers number (Z): $\pm 1, \pm 2, \pm 3, \pm 4, \ldots$ Rational numbers (Q): a/b: $a, b \in Z$, $b \neq 0$ Real numbers (R): d. all numbers known. Complex numbers(): e. Involve all numbers \subset \subset \subset \subset Ζ 0 Ν R С Natural Integers Rationa Real Complex

Example:

1) Let P(x,y) be the predicate " x.y = 1 " where x and y assume any real numbers.

1

 $\forall x \exists y P(x,y)$: "for all x, there exists a value of y such that x.y = 1" Is false (0).

~ $[\forall x \exists y P(x,y)] = \exists x \forall y (~ P(x,y)) = \exists x \forall y [x.y \neq 1] \text{ is true } (1).$

2) Let Q (x) be the predicate " x is prime integer " and R (x) be the predicate " x² is odd integer " and let the universe of discourse be the set of integer.

 $\forall x [Q(x) \Rightarrow R(x)]$: "for all values of x, if x is prime integer, then x² i odd integer " is false (0).

- $\sim [\forall x [Q(x) \Rightarrow R(x)]]$
- $\Rightarrow \exists x \sim [(Q(x) \Rightarrow R(x))]$
- → $\exists x \sim [\sim (Q(x) \lor R(x))]$
- $\Rightarrow \exists x [(Q(x) \land \neg R(x)]$
- → "there exists a value of x such that x is prime integer and x² is even" Is true (1).

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