Department of Mathematics
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Discrete Mathematics

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Chapter one

LOGIC

Section 1.4
Predicate.
X is even integer : not proposition.
It becomes a proposition only when the value of has been determined.
Set x=4, then we have a true proposition
Set x=5, then we have a false proposition

An assertion that contains one or more variables is a predicate
P(x), P(x,y): is a predicate.
P(2): is a proposition.
P(5): is a proposition.

The set that we choose the values of x from it is called the **University of Discourse**.

Example:
Let the **Universe of discourse** be the set of integers, and let P(x,y) be the predicate "x.y = 12"

P (1,3): "1.3=12": "3=12" false proposition
P (2,6): "2.6=12": "12=12" true proposition

Example:
Let Q(x) be the predicate "2.x is even integer" and the **universe of discourse** be the set of integers.
Q(x) is true for all x in the universe of discourse
$\forall$x Q(x) : for all x in the universe of discourse Q(x) hold.
$\forall$x Q(x) : is a proposition.

Example:
Q (x) : 2.x is even integers, universe of discourse all integers.
Q (1), Q (2), Q (9), Q (11) : is true.
NOTE: that for all values of **x**, Q (x) is true.

Clarification:
A: "for all values of x, Q (x) is true OR hold"
A: is true proposition.
In general: "for all values of x, P (x) is true" is proposition.
We denote for all by $\forall$
A: $\forall$x Q(x)
A: for all values of x, Q (x) is true

Example:
Let Q (x) be the predicate "x.1= x", and universe of discourse be all real numbers.
$\forall$x Q (x) : "for all values of x , x.1= x". is true proposition

Example:
Let P (x) be the predicate "x.1= 0", and universe of discourse be all real numbers.
$\forall$x Q(x) : "for all values of x , x.1= 0". is false proposition
But there is a value of x such that P (x) hold.

We denote There exists : / There is : By $\exists$

$\exists$x P (x):"there exists a value of x such that P (x) hold" is true proposition

Example:
Let the universe of discourse be the real numbers.

P (x) : " $x^2 \geq 0$ "
Q (x) : " 3.x > 10"

1. $\forall$x P (x): "for all values of x, $x^2 \geq 0$ " True (1)
2. $\forall$x Q (x): "for all values of x, 3.x > 10" False (0)
3. $\exists$x P (x) : " there exists a value of x such that $x^2 \geq 0$ " True (1)
4. $\exists$x Q (x) : " there exists a value of x such that 3.x > 10" True (1)

Example:
Let P (x,y) be the predicate "x > y" and let the universe of discourse be the set of real numbers.

1. $\exists$x $\forall$y P (x,y): "there exists a value of x such that for all values of y,
x > y → is false (0).
2. $\exists$x $\exists$y P (x,y): "there exists a value of x and a values of y,
x > y → is true (1).
3. $\forall$x $\exists$y P (x,y): "for all values of x, there exists a value of y,
Such that x > y → is true (1).

Example:
Let the universe of discourse be the set integers.
a. $\forall x\ \exists y\ [\ x + y = 0\ ]$: "for all x, there exists a value of y such that $x + y = 0$ " → is true (1).
b. $\exists x\ \forall y\ [\ x.y = 0\ ]$: " there exists value of x, such that for all of y, $x.y = 0$ " → is true (1)
c. $\exists x\ \exists y\ [\ x.y = 15\ ]$: "there exists value of x, and a value of y such that $x.y = 15$" → is true (1).
d. $\forall x\ \forall y\ [\ x.y > 0\ ]$: " for all x and for all y, $x.y > 0$" → is false (0).

Example:
P (x,y) : " $x.y = 4$ "

$\forall x\ \exists y\ P(x,y)$: "for all x, there exists a value of y such that $x.y = 4$"
→ is false (0).

NOW:
If P (x) is a predicate then the negation of $\forall x\ P(x)$ and $\exists x\ P(x)$ is gives by the following :
$\sim [\ \forall x\ P(x)\ ] = \ \exists x\ (\sim P(x))$
$\sim [\ \exists x\ P(x)\ ] = \ \forall x\ (\sim P(x))$

Example:
Let Q (x) be the predicate " $x^2 > 0$ " and let the universe of discourse be the set of integers.
$\forall x\ Q(x)$: is false (0).
$\sim (\ \forall x\ Q(x)\ )$: is true (1).
That's mean: $\exists x\ (\sim Q(x)) \rightarrow (\ \exists x\ [x^2 \leq 0\ ])$

- Type of numbers
  a. Natural number (N):
  1, 2, 3, 4, ………………….
  b. Integers number (Z):
  0, ±1, ±2, ±3, ±4, …………
  c. Rational numbers (Q):
  a/b : a, b ∈ Z , b ≠ 0
  d. Real numbers (R):
  all numbers known.
  e. Complex numbers():
  Involve all numbers

N	⊂	Z	⊂	Q	⊂	R	⊂	C
Natural		Integers		Rationa		Real		Complex

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Example:

1) Let P (x,y) be the predicate " x.y = 1 " where x and y assume any real numbers.

$\forall x \; \exists y \; P(x,y)$: "for all x, there exists a value of y such that x.y = 1" Is false (0).

$\sim [\forall x \; \exists y \; P(x,y)] = \exists x \; \forall y \; (\sim P(x,y)) = \exists x \; \forall y \; [x.y \neq 1]$ is true (1).

2) Let Q (x) be the predicate " x is prime integer " and R (x) be the predicate " x^2 is odd integer " and let the universe of discourse be the set of integer.

$\forall x \; [Q(x) \Rightarrow R(x)]$: " for all values of x, if x is prime integer, then x^2 is odd integer " is false (0).

$\sim [\forall x \; [Q(x) \Rightarrow R(x)]]$

→ $\exists x \sim [(Q(x) \Rightarrow R(x))]$

→ $\exists x \sim [\sim (Q(x) \vee R(x))]$

→ $\exists x \; [(Q(x) \wedge \sim R(x)]$

→ "there exists a value of x such that x is prime integer and x^2 is even" Is true (1).