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Discrete Mathematics

Yarmouk University

Second Semester

2009/2010

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Chapter one

LOGIC

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Section 1.2:
The conditional and Biconditional.

1. The connective " **implies** OR **if then**" denoted by ($\Rightarrow$), this connective is called **the conditional connective**.

Example:

P: I graduate.

Q: I will get a job.

$P \Rightarrow Q$, if graduate, then I will get a job.

- A and B and $A \Rightarrow B$.

Truth table (2^2)

P	**Q**	**$P \Rightarrow Q$**
1	1	1
1	0	0
0	1	1
0	0	1

- We use $A \Rightarrow B$ to mean that wherever A is true, then B must be true.
- NOTE:

That the only way to have $A \Rightarrow B$ false that A is true but B is false.

$P \Rightarrow Q$ **equivalent** $\sim P \vee Q$.

PROOF:

Truth table (2^2)

P	Q	~ P	~ Q	**$P \Rightarrow Q$**	**$\sim P \vee Q$**
1	1	0	0	**1**	**1**
1	0	0	1	**0**	**0**
0	1	1	0	**1**	**1**
0	0	1	1	**1**	**1**

- We can use (table) to prove that $\sim (P \Rightarrow Q)$ equivalent to $(P \wedge \sim Q)$.

PROOF:

Truth table (2^2)

P	Q	~ P	~ Q	$P \Rightarrow Q$	**$\sim (P \Rightarrow Q)$**	**$(P \wedge \sim Q)$**
1	1	0	0	1	**0**	**0**
1	0	0	1	0	**1**	**1**
0	1	1	0	1	**0**	**0**
0	0	1	1	1	**0**	**0**

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- Contrapositive Law

$P \Rightarrow Q$ equivalent $\sim Q \Rightarrow \sim P$.
PROOF:
Truth table (2^2)

P	**Q**	**~ P**	**~ Q**	**$P \Rightarrow Q$**	**$\sim P \Rightarrow \sim Q$**
1	1	0	0	**1**	**1**
1	0	0	1	**0**	**0**
0	1	1	0	**1**	**1**
0	0	1	1	**1**	**1**

- $P \Rightarrow Q$ NOT equivalent $Q \Rightarrow P$.

PROOF:
Truth table (2^2)

P	**Q**	**$P \Rightarrow Q$**	**$Q \Rightarrow P$**
1	1	**1**	**1**
1	0	**0**	**1**
0	1	**1**	**0**
0	0	**1**	**1**

Then $P \Rightarrow Q$ **NOT** equivalent $Q \Rightarrow P$

2. The connective **if and only if** denoted by $\Leftrightarrow$, this connective called **Biconditional.**

Example:
P = 4 is even number.
Q = 4 is divisible by 2.
$((P \Rightarrow Q) \wedge (Q \Rightarrow P)) \Leftrightarrow (P \Leftrightarrow Q)$.
PROOF:
Truth table (2^2)

P	**Q**	**$P \Rightarrow Q$**	**$Q \Rightarrow P$**	**$((P \Rightarrow Q) \wedge (Q \Rightarrow P))$**	**$P \Leftrightarrow Q$**
1	1	1	1	**1**	**1**
1	0	0	1	**0**	**0**
0	1	1	0	**0**	**0**
0	0	1	1	**1**	**1**

- NOTE:

That $P \Leftrightarrow Q$ is true when P and Q have the same truth values.

- Two forms A and B are equivalent if $A \Leftrightarrow B$ is true.
- $P \Rightarrow Q$ equivalent $\sim P \vee Q$.

$(P \Rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

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DEFITIONS:

A proposition that is always true is called **A tautology**

Example:

$(P \vee \sim P)$: is a tautology.

Truth table (2^1)

P	~ P	P ∨ ~ P
1	0	1
0	1	1

All result in truth table is true, then is **A tautology**.

- A equivalent B

if A $\Leftrightarrow$ B is a tautology.

DEFITIONS:

A proposition that is always false is called **Contradiction**.

$(P \wedge \sim P)$: is a contradiction.

Truth table (2^1)

P	~ P	P ∧ ~ P
1	0	0
0	1	0

All result in truth table is false, then is **A contradiction**.

Example:

$(P \Rightarrow Q) \Leftrightarrow (P \wedge \sim Q)$ is a contradiction.

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IMPORTANT LOGICAL IDENTITIES.

1. Idempotent Laws.
   1.1. $P \Leftrightarrow (P \vee P)$
   1.2. $P \Leftrightarrow (P \wedge P)$

2. Double Negation
   2.1. $(\sim(\sim P)) \Leftrightarrow P$

3. Commutativity
   3.1. $(P \vee Q) \Leftrightarrow (Q \vee P)$
   3.2. $(P \wedge Q) \Leftrightarrow (Q \wedge P)$

4. Distributive Laws
   4.1. $(P \wedge (Q \vee R)) \Leftrightarrow ((P \wedge Q) \vee (P \wedge Q))$
   4.2. $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee Q))$

5. Associativity
   5.1. $(P \vee (Q \vee R)) \Leftrightarrow ((P \vee Q) \vee R)$
   5.2. $(P \wedge (Q \wedge R)) \Leftrightarrow ((P \wedge Q) \wedge R)$

6. DeMorgan's Laws
   6.1. $(\sim(P \vee Q)) \Leftrightarrow (\sim P \wedge \sim Q)$
   6.2. $(\sim(P \wedge Q)) \Leftrightarrow (\sim P \vee \sim Q)$

7. Implication
   7.1. $(P \Rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

8. Contrapositive
   8.1. $(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$

In the identities that follow, we shall denote a proposition that is always true by 1 and a proposition that always false by 0.

9. Laws Group One
   9.1. $(P \vee 0) \Leftrightarrow P$
   9.2. $(P \wedge 1) \Leftrightarrow P$

10. Laws Group Two
   10.1. $(P \vee 1) \Leftrightarrow 1$
   10.2. $(P \wedge 0) \Leftrightarrow 0$

11. Laws Group Three
   11.1. $(P \vee \sim P) \Leftrightarrow 1$
   11.2. $(P \wedge \sim P) \Leftrightarrow 0$

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PROOF:

Idempotent laws.

P ⇔ (P ∨ P)

Truth table

P	**P ∨ P**	**P ⇔ (P ∨ P)**
1	1	1
0	0	1

P ⇔ (P ∧ P)

Truth table

P	**P ∧ P**	**P ⇔ (P ∧ P)**
1	1	1
0	0	1

double negation

(~ (~ P)) ⇔ P

Truth table

P	**~P**	**~ (~P)**	**(~ (~ P)) ⇔ P**
1	0	**1**	**1**
0	1	**0**	**1**

Commutativity

(P ∨ Q) ⇔ (Q ∨ P)

Truth table

P	**Q**	**P ∨ Q**	**Q ∨ P**	**(P ∨ Q) ⇔ (Q ∨ P)**
1	1	1	1	1
1	0	1	1	1
0	1	1	1	1
0	0	0	0	1

(P ∧ Q) ⇔ (Q ∧ P)

Truth table

P	**Q**	**P ∧ Q**	**Q ∧ P**	**(P ∧ Q) ⇔ (Q ∧ P)**
1	1	1	1	1
1	0	0	0	1
0	1	0	0	1
0	0	0	0	1

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distributive laws

$(P \wedge (Q \vee R)) \Leftrightarrow ((P \wedge Q) \vee (P \wedge R))$

P	Q	R	Q∨R	P∧(Q∨R)	P∧Q	P∧R	((P∧Q)∨(P∧R))	(P∧(Q∨R)) ⇔ ((P∧Q)∨(P∧R))
1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1	1
1	0	1	1	1	0	1	1	1
1	0	0	0	0	0	0	0	1
0	1	1	1	0	0	0	0	1
0	1	0	1	0	0	0	0	1
0	0	1	1	0	0	0	0	1
0	0	0	0	0	0	0	0	1

$(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$

P	Q	R	Q∧R	P∨(Q∧R)	P∨Q	P∨R	((P∨Q)∧(P∨R))	(P∨(Q∧R)) ⇔ ((P∨Q)∧(P∨R))
1	1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1	1
1	0	1	0	1	1	1	1	1
1	0	0	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0	1
0	0	1	0	0	0	1	0	1
0	0	0	0	0	0	0	0	1

associativity

$(P \vee (Q \vee R)) \Leftrightarrow ((P \vee Q) \vee R)$

Truth table

P	Q	R	Q∨R	P∨(Q∨R)	P∨Q	(P∨Q)∨R	(P∨(Q∨R)) ⇔ ((P∨Q)∨R)
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	1	1	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	0	1	1	1	0	1	1
0	0	0	0	0	0	0	1

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$(P \wedge (Q \wedge R)) \Leftrightarrow ((P \wedge Q) \wedge R)$

Truth table

P	Q	R	$Q \wedge R$	$P \wedge (Q \wedge R)$	$P \wedge Q$	$(P \wedge Q) \wedge R$	$(P \wedge (Q \wedge R)) \Leftrightarrow ((P \wedge Q) \wedge R)$
1	1	1	1	1	1	1	1
1	1	0	0	0	1	0	1
1	0	1	0	0	0	0	1
1	0	0	0	0	0	0	1
0	1	1	1	0	0	0	1
0	1	0	0	0	0	0	1
0	0	1	0	0	0	0	1
0	0	0	0	0	0	0	1

DeMorgan's laws

$(\sim (P \vee Q)) \Leftrightarrow (\sim P \wedge \sim Q)$

Truth table

P	Q	$P \vee Q$	$\sim (P \vee Q)$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$(\sim (P \vee Q)) \Leftrightarrow (\sim P \wedge \sim Q)$
1	1	1	0	0	0	0	1
1	0	1	0	0	1	0	1
0	1	1	0	1	0	0	1
0	0	0	1	1	1	1	1

$(\sim (P \wedge Q)) \Leftrightarrow (\sim P \vee \sim Q)$

Truth table

P	Q	$P \wedge Q$	$\sim (P \wedge Q)$	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$(\sim (P \wedge Q)) \Leftrightarrow (\sim P \vee \sim Q)$
1	1	1	0	0	0	0	1
1	0	0	1	0	1	1	1
0	1	0	1	1	0	1	1
0	0	0	1	1	1	1	1

implication

$(P \Rightarrow Q) \Leftrightarrow (\sim P \vee Q)$

Truth table

P	Q	$P \Rightarrow Q$	$\sim P$	$\sim P \vee Q$	$(P \Rightarrow Q) \Leftrightarrow (\sim P \vee Q)$
1	1	1	0	1	1
1	0	0	0	0	1
0	1	1	1	1	1
0	0	1	1	1	1

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Contrapositive

$(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$

Truth table

P	Q	P ⇒ Q	~ P	~ Q	~ Q ⇒ ~ P	(P ⇒ Q) ⇔ (~ Q ⇒ ~ P)
1	1	1	0	0	1	1
1	0	0	0	1	0	1
0	1	1	1	0	1	1
0	0	1	1	1	1	1

In the identities that follow, we shall denote a proposition that is always true by 1 and a proposition that always false by 0.

laws group one

$(P \vee 0) \Leftrightarrow P$

Truth table

P	VALUE	(P ∨ 0)	(P ∨ 0) ⇔ P
1	0	1	1
0	0	0	1

$(P \wedge 1) \Leftrightarrow P$

Truth table

P	VALUE	(P ∧ 1)	(P ∧ 1) ⇔ P
1	1	1	1
0	1	0	1

laws group two

$(P \vee 1) \Leftrightarrow 1$

Truth table

P	VALUE	(P ∨ 1)	(P ∨ 1) ⇔ 1
1	1	1	1
0	1	1	1

$(P \wedge 0) \Leftrightarrow 0$

Truth table

P	VALUE	(P ∧ 0)	(P ∧ 0) ⇔ 0
1	0	0	1
0	0	0	1

laws group three

$(P \vee \sim P) \Leftrightarrow 1$

Truth table

P	~ P	VALUE	(P ∨ ~ P)	(P ∨ ~ P) ⇔ 1
1	0	1	1	1
0	1	1	1	1

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$(P \wedge \sim P) \Leftrightarrow 0$

Truth table

P	**$\sim P$**	**VALUE**	**$(P \wedge \sim P)$**	**$(P \wedge \sim P) \Leftrightarrow 0$**
1	0	0	0	1
0	1	0	0	1

Negate the following in such a way that the symbol $\Rightarrow$ does not appear.

i. $P \Rightarrow (Q \wedge R)$

ii. $(P \wedge Q) \Rightarrow R$

iii. $(P \Rightarrow Q) \Rightarrow R$

Solve problem:

- $P \Rightarrow (Q \wedge R)$

$\sim (P \Rightarrow (Q \wedge R))$

$\Leftrightarrow \sim (\sim P \vee (Q \wedge R))$

$\Leftrightarrow P \wedge (\sim (Q \wedge R))$

$\Leftrightarrow P \wedge (\sim Q \vee \sim R)$

- $(P \wedge Q) \Rightarrow R$

$\sim ((P \wedge Q) \Rightarrow R)$

$\Leftrightarrow \sim (\sim (P \wedge Q) \vee R))$

$\Leftrightarrow (P \wedge Q) \wedge \sim R$

$\Leftrightarrow P \wedge Q \wedge \sim R$

- $(P \Rightarrow Q) \Rightarrow R$

$\sim ((P \Rightarrow Q) \Rightarrow R)$

$\Leftrightarrow \sim ((\sim P \vee Q) \Rightarrow R)$

$\Leftrightarrow \sim (\sim (\sim p \vee Q) \vee R)$

$\Leftrightarrow \sim ((P \vee Q)) \wedge \sim R)$

$\Leftrightarrow (\sim (P \vee Q) \vee R)$

$\Leftrightarrow \sim P \wedge \sim Q \vee R$

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