Department of Mathematics
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Discrete Mathematics

Yarmouk University

Second Semester

2009/2010

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Chapter one

LOGIC

Section 1.1

Propositions.

DEFINITION:

A proposition is a declarative sentence to which we can assign a truth value of either true or false but not both.

Examples:

1. 4 is even number (true) (1).
2. 5 is even number (false) (0).

We will use A, B, C, P, Q, R, to present propositions there letters called **logical variables.**

- Compound proposition.

We will use connectives to form compound propositions.

1. To connectives **AND** denoted by $\underline{\wedge}$.
2. To connectives **OR** denoted by $\underline{\vee}$.
3. To connectives **NOT** denoted by $\underline{\sim}$.

- A, B variables:
  1. $(A \wedge B)$ …. **Conjunction** of A and B.
  2. $(A \vee B)$ …. **Disjunction** of A and B.

Examples:

1. Read this statements and answer following questions.

questions	Answers
A: the earth is round.	1 (true)
B: the sun is cold.	0 (false)
C: it rains in Spain.	1 (true)

questions	Answers
$A \wedge B$: the earth is round and the sun is cold.	0 (false)
$B \vee C$: the sun is cold or it rains in Spain.	1 (true)
$\sim B$: the sun is not cold.	1 (true)

P and ~P always have apposite truth value.

Truth table (2^1).

P	**~P**
1	0
0	1

- A and B and A ∧ B.

Truth table (2^2).

A	**B**	**A ∧ B**
1	1	1
1	0	0
0	1	0
0	0	0

- A and B and A ∨ B.

Truth table (2^2).

A	**B**	**A ∨ B**
1	1	1
1	0	1
0	1	1
0	0	0

- A ∧ (B ∨ C).

Truth table (2^3).

A	**B**	**C**	**B ∨ C**	**A ∧ (B ∨ C)**
1	1	1	1	1
1	1	0	1	1
1	0	1	1	1
1	0	0	0	0
0	1	1	1	0
0	1	0	1	0
0	0	1	1	0
0	0	0	0	0

- If two forms have **identical truth table** then said to be **equivalent**.
- If A and B are **equivalent** forms, we write A=B and called **logically identical**.

- **LAWS:**
  - **DeMorgan's laws.**
    1. $\sim (A \wedge B) = \sim A \vee \sim B$
    2. $\sim (A \vee B) = \sim A \wedge \sim B$

PROOF:

1. $\sim (A \wedge B) = \sim A \vee \sim B$

Truth table (2^2).

A	B	A ∧ B	~ (A ∧ B)	~A	~B	~ A ∨ ~ B
1	1	1	**0**	0	0	**0**
1	0	0	**1**	0	1	**1**
0	1	0	**1**	1	0	**1**
0	0	0	**1**	1	1	**1**

2. $\sim (A \vee B) = \sim A \wedge \sim B$

Truth table (2^2).

A	B	A ∨ B	~ (A ∨ B)	~A	~B	~ A ∧ ~ B
1	1	1	**0**	0	0	**0**
1	0	1	**0**	0	1	**0**
0	1	1	**0**	1	0	**0**
0	0	0	**1**	1	1	**1**

- **Distributive laws**
  1. $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$
  2. $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

PROOF:

1. $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$

Truth table (2^3)

A	B	C	B ∨ C	A∧(B∨C)	A∧B	A∧C	(A ∧ B)∨(A ∧ C)
1	1	1	1	**1**	1	1	**1**
1	1	0	1	**1**	1	0	**1**
1	0	1	1	**1**	0	1	**1**
1	0	0	0	**0**	0	0	**0**
0	1	1	1	**0**	0	0	**0**
0	1	0	1	**0**	0	0	**0**
0	0	1	1	**0**	0	0	**0**
0	0	0	0	**0**	0	0	**0**

2. $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

Truth table (2^3)

A	**B**	**C**	**B∧C**	**A∨(B∧C)**	**A∨B**	**A∨C**	**(A∨B)∧(A∨C)**
1	1	1	1	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
0	1	1	1	**1**	1	1	**1**
0	1	0	0	**0**	1	0	**0**
0	0	1	0	**0**	0	1	**0**
0	0	0	0	**0**	0	0	**0**