

**Department of Statistics  
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SATS 101

Introduction to Probability  
and Statistics

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## Chapter 9

### Large-Sample Tests of Hypotheses

#### Introduction

- Suppose that a pharmaceutical company is concerned that the mean potency  $\mu$  of an antibiotic meet the minimum government potency standards. They need to decide between two possibilities:
  - The mean potency  $\mu$  does not exceed the mean allowable potency.
  - The mean potency  $\mu$  exceeds the mean allowable potency.
- This is an example of a test of hypothesis.
- Similar to a courtroom trial. In trying a person for a crime, the jury needs to decide between one of two possibilities:
  - The person is guilty.
  - The person is innocent.
- To begin with, the person is assumed innocent.
- The prosecutor presents evidence, trying to convince the jury to reject the original assumption of innocence, and conclude that the person is guilty.

#### Parts of a Statistical Test

1. The null hypothesis,  $H_0$ :
  - Assumed to be true until we can prove otherwise.
2. The alternative hypothesis,  $H_a$ :
  - Will be accepted as true if we can disprove  $H_0$

#### Court trial:

$H_0$ : innocent

$H_a$ : guilty

#### Pharmaceuticals:

$H_0$ :  $m$  does not exceeds allowed amount

$H_a$ :  $m$  exceeds allowed amount

3. The test statistic and its  $p$ -value:
  - A single statistic calculated from the sample which will allow us to reject or not reject  $H_0$ , and
  - A probability, calculated from the test statistic that measures whether the test statistic is likely or unlikely, assuming  $H_0$  is true.
4. The rejection region:
  - A rule that tells us for which values of the test statistic, or for which  $p$ -values, the null hypothesis should be rejected.
5. Conclusion:
  - Either “Reject  $H_0$ ” or “Do not reject  $H_0$ ”, along with a statement about the reliability of your conclusion.

How do you decide when to reject  $H_0$ ?

- Depends on the significance level,  $\alpha$ , the maximum tolerable risk you want to have of making a mistake, if you decide to reject  $H_0$ .
- Usually, the significance level is  $\alpha = .01$  or  $\alpha = .05$ .

### Example

- The mayor of a small city claims that the average income in his city is \$35,000 with a standard deviation of \$5000. We take a sample of 64 families, and find that their average income is \$30,000. Is his claim correct?

1-2. We want to test the hypothesis:

$H_0: \mu = 35,000$  (mayor is correct) versus

$H_a: \mu \neq 35,000$  (mayor is wrong)

Start by assuming that  $H_0$  is true and  $\mu = 35,000$ .

3. The best estimate of the population mean  $m$  is the sample mean, \$30,000:
  - From the Central Limit Theorem the sample mean has an approximate normal distribution with mean  $\mu = 35,000$  and standard error  $SE = 5000/8 = 625$ .
  - The sample mean, \$30,000 lies  $z = (30,000 - 35,000)/625 = -8$  standard deviations below the mean.
  - The probability of observing a sample mean this far from  $\mu = 35,000$  (assuming  $H_0$  is true) is *nearly zero*.
4. From the Empirical Rule, values more than three standard deviations away from the mean are considered extremely unlikely. Such a value would be extremely unlikely to occur if indeed  $H_0$  is true, and would give reason to reject  $H_0$ .
5. Since the observed sample mean, \$30,000 is so unlikely, we choose to reject  $H_0: \mu = 35,000$  and conclude that the mayor's claim is incorrect.
6. The probability that  $\mu = 35,000$  and that we have observed such a small sample mean just by chance is *nearly zero*.

### Large Sample Test of a Population Mean, $\mu$

- Take a random sample of size  $n \geq 30$  from a population with mean  $m$  and standard deviation  $s$ .
- We assume that either
  1.  $s$  is known or
  2.  $s \approx \hat{s}$  since  $n$  is large
- The hypothesis to be tested is
  1.  $H_0: \mu = \mu_0$  versus  $H_a: \mu \neq \mu_0$

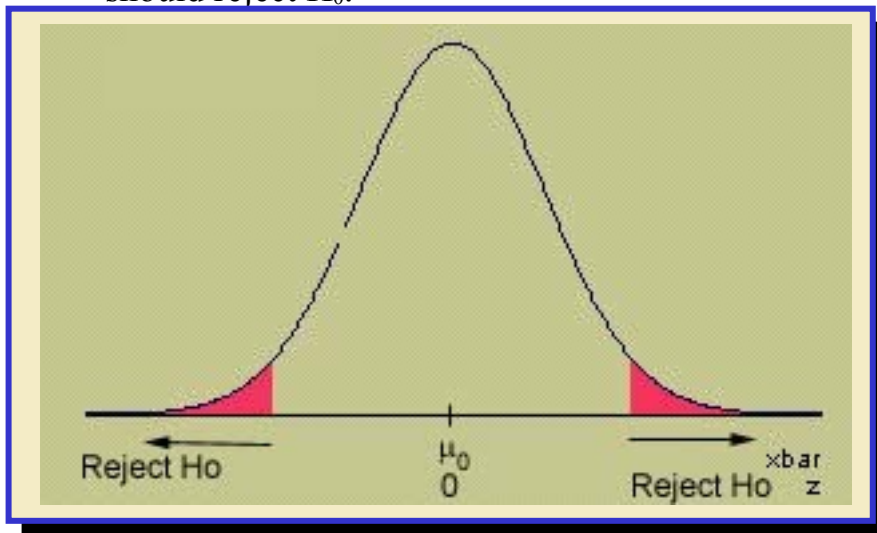
### Test Statistic

- Assume to begin with that  $H_0$  is true. The sample mean is our best estimate of  $\mu$ , and we use it in a standardized form as

the test statistic: 
$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \approx \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$$

Since  $\bar{x}$  has an approximate normal distribution with mean  $\mu_0$  and standard deviation  $\sigma / \sqrt{n}$ .

- If  $H_0$  is true the value of  $\bar{x}$  should be close to  $\mu_0$ , and  $z$  will be close to 0. If  $H_0$  is false,  $\bar{x}$  will be much larger or smaller than  $\mu_0$ , and  $z$  will be much larger or smaller than 0, indicating that we should reject  $H_0$ .



### Likely or Unlikely?

- Once you've calculated the observed value of the test statistic, calculate its **p-value**:

**p-value:** The probability of observing, *just by chance*, a test statistic as extreme or even more extreme than what we've actually observed. If  $H_0$  is rejected this is the actual probability that we have made an incorrect decision.

- If this probability is very small, less than some **pre assigned significance level**,  $\alpha$ ,  $H_0$  can be rejected.

### Example

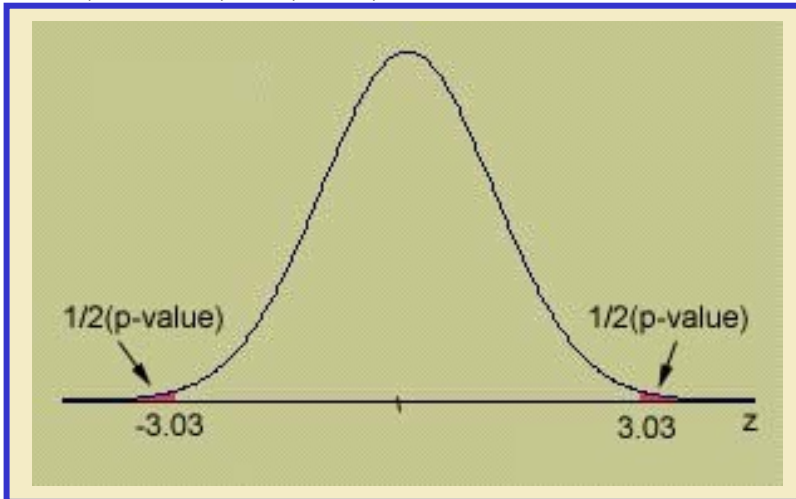
- The daily yield for a chemical plant has averaged 880 tons for several years.

The quality control manager wants to know if this average has changed. She randomly selects 50 days and records an average yield of 871 tons with a standard deviation of 21 tons.

$$\begin{aligned} H_0 : \mu &= 880 & \text{Test statistic :} \\ H_a : \mu &\neq 880 & z \approx \frac{\bar{x} - \mu_0}{s / \sqrt{n}} = \frac{871 - 880}{21 / \sqrt{50}} = -3.03 \end{aligned}$$

What is the probability that this test statistic or something even more extreme (far from what is expected if  $H_0$  is true) could have happened *just by chance*?

$$\begin{aligned} p\text{-value} &: P(z > 3.03) + P(z < -3.03) \\ &= 2P(z < -3.03) = 2(.0012) = .0024 \end{aligned}$$



This is an unlikely occurrence, which happens about 2 times in 1000, assuming  $\mu = 880$ !

- To make our decision clear, we choose a significance level, say  $\alpha = .01$ .

If the  $p$ -value is less than  $\alpha$ ,  $H_0$  is rejected as false. You report that the results are statistically significant at level  $\alpha$ .

If the  $p$ -value is greater than  $\alpha$ ,  $H_0$  is not rejected. You report that the results are not significant at level  $\alpha$ .

Since our  $p$ -value = .0024 is less than  $\alpha$ , we reject  $H_0$  and conclude that the average yield has changed.

## Using a Rejection Region

If  $\alpha = .01$ , what would be the **critical value** that marks the “dividing line” between “not rejecting” and “rejecting”  $H_0$ ?

If  $p\text{-value} < \alpha$ ,  $H_0$  is rejected.

If  $p\text{-value} > \alpha$ ,  $H_0$  is not rejected.

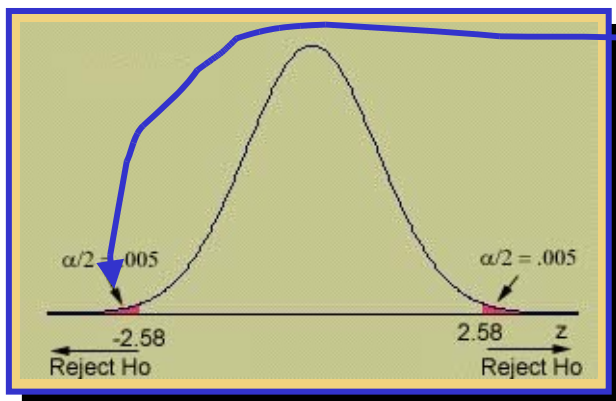
The dividing line occurs when  $p\text{-value} = \alpha$ . This is called the **critical value** of the test statistic.

Test statistic  $>$  critical value implies  $p\text{-value} < \alpha$ ,  $H_0$  is rejected.

Test statistic  $<$  critical value implies  $p\text{-value} > \alpha$ ,  $H_0$  is not rejected.

## Example

What is the critical value of  $z$  that cuts off exactly  $\alpha/2 = .01/2 = .005$  in the tail of the  $z$  distribution?



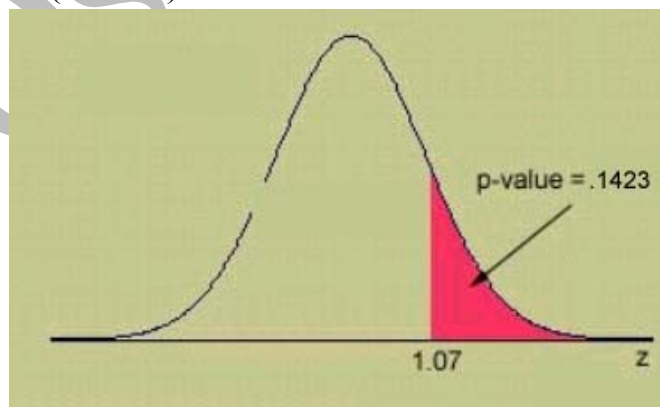
For our example  $z = -3.03$  falls in the rejection region and  $H_0$  is rejected at the 1% significance level.

**Rejection Region:** Reject  $H_0$  if  $z > 2.58$  or  $z < -2.58$ . If the test statistic falls in the rejection region, its  $p\text{-value}$  will be less than  $\alpha = .01$ .

## $p\text{-Value Approach}$

- The probability that our sample results or something even more unlikely would have occurred *just by chance*, when  $m = 250,000$ .

$$p\text{-value} : P(z > 1.07) = 1 - .8577 = .1423$$



Since the  $p\text{-value}$  is greater than  $\alpha = .01$ ,  $H_0$  is not rejected. There is insufficient evidence to indicate that  $m$  is greater than \$250,000.