Department of Statistics
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SATS 101

Introduction to Probability and Statistics

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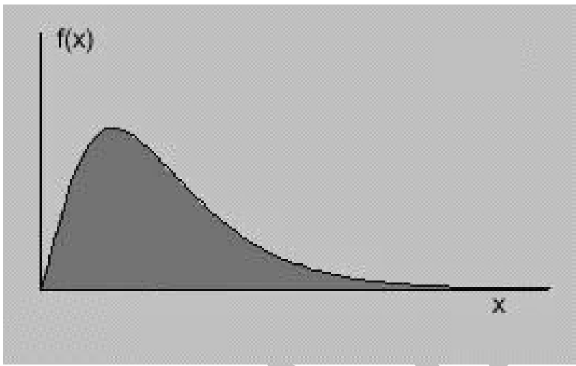
Chapter 6

The Normal Probability Distribution

Continuous Random Variables

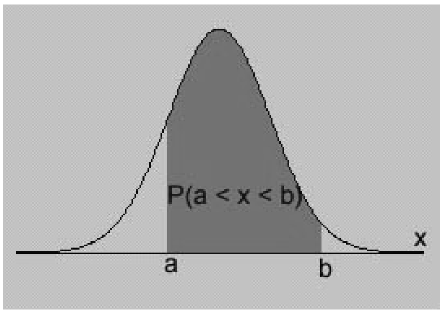
- Continuous random variables can assume the infinitely many values corresponding to points on a line interval.
- **Examples:**
  - Heights, weights
  - length of life of a particular product
  - experimental laboratory error
- A **smooth curve** describes the probability distribution of a continuous random variable.



- The depth or density of the probability, which varies with *x*, may be described by a mathematical formula $f(x)$, called the **probability distribution** or **probability density function** for the random variable *x*.

Properties of Continuous Probability Distributions

- The area under the curve is equal to **1.**
- $P(a \le x \le b)$ = **area under the curve** between a and b.
- There is no probability attached to any single value of *x*. That is, $\mathbf{P(x = a) = 0}$.



Continuous Probability Distributions

- There are many different types of continuous random variables
- We try to pick a model that
  - Fits the data well
  - Allows us to make the best possible inferences using the data.
- One important continuous random variable is the **normal random variable**.

The Normal Distribution

- The formula that generates the normal probability distribution is:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \text{for } -\infty < x < \infty$$

$$e = 2.7183 \qquad \pi = 3.1416$$

μ and σ are the population mean and standard deviation.

- The shape and location of the normal curve changes as the mean and standard deviation change.

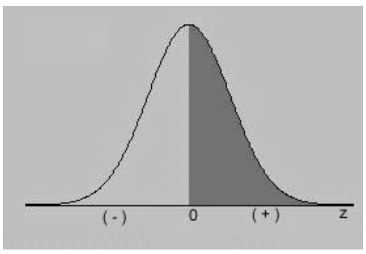
The Standard Normal Distribution

- To find P(a < x < b), we need to find the area under the appropriate normal curve.
- To simplify the tabulation of these areas, we **standardize** each value of x by expressing it as a z-score, the number of standard deviations s it lies from the mean m.

$$z = \frac{x-\mu}{\sigma}$$

The Standard Normal (z) Distribution

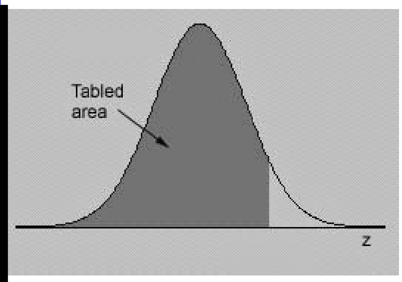
- Mean = 0; Standard deviation = 1
- When $x = \mu$, $z = 0$
- Symmetric about $z = 0$
- Values of z to the left of center are negative
- Values of z to the right of center are positive
- Total area under the curve is 1.



Using Table 3

The four digit probability in a particular row and column of Table 3 gives the area under the *z* curve to the left that particular value of *z*.

z	.00	.01	.02	.03	.04	.05	.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	.5636
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	.6026
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	.6406
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	.6772
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051
0.9	0.8159	0.8186	0.8212	0.8328	0.8264	0.8289	0.8315
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770
1.2	[illegible]	[illegible]	0.8888	0.8907	0.8925	0.8944	0.8962
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	[illegible]



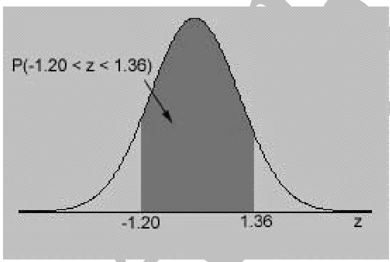
Area for $z = 1.36$

Example

Use Table 3 to calculate these probabilities:

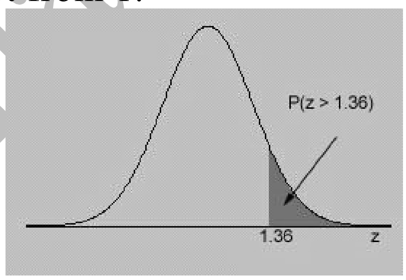
✓ To find the area between two values of *z*, find the two areas in Table 3, and subtract one from the other.

$P(-1.20 \leq z \leq 1.36)$ = .9131 - .1151 = .7980



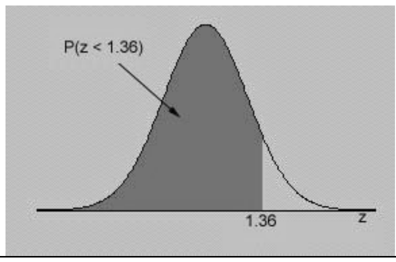
✓ To find an area to the right of a *z*-value, find the area in Table 3 and subtract from 1.

) P(*z* >1.36
0869. = 9131. - 1 =



✓ To find an area to the left of a *z*-value, find the area directly from the table.

$P(z \leq 1.36)$ = .9131



Remember the Empirical Rule: Approximately 95% of the measurements lie within 2 standard deviations of the mean.

Remember the Empirical Rule: Approximately 99.7% of the measurements lie within 3 standard deviations of the mean.

)P(-3 ≤ z ≤ 3
9974.=0013. - 9987. =