**Department of Statistics**
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**Yarmouk University**

SATS 101

# Introduction to Probability and Statistics

# Yarmouk University

## Second Semester

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Chapter 5

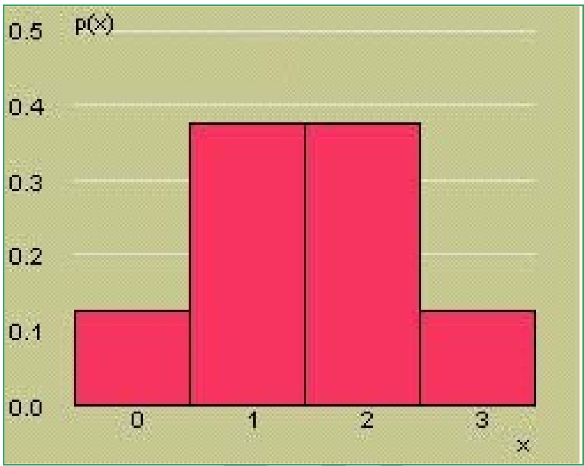
Several Useful Discrete Distributions

## Introduction

- Discrete random variables take on only a finite or countable number of values.
- Three discrete probability distributions serve as models for a large number of practical applications:
- **The binomial random variable**
- **The Poisson random variable**
- **The HyperGeometric random variable**

## The Binomial Random Variable

- The **coin-tossing experiment** is a simple example of a **binomial random variable.** Toss a fair coin $n = 3$ times and record $x$ = number of heads.




| $x$ | $p(x)$ |
|---|---|
| 0 | 1/8 |
| 1 | 3/8 |
| 2 | 3/8 |
| 3 | 1/8 |

- Many situations in real life resemble the coin toss, but the coin is not necessarily fair, so that P(H) ≠ 1/2.
- **Example:** A geneticist samples 10 people and counts the number who have a gene linked to Alzheimer's disease.
- **Coin: Person**
- **Head: Has gene**
- **Tail: Doesn't have gene**
- **Number of tosses: $n$ = 10**
- **P(H): P(has gene) = proportion in the population who have the gene.**

**The Binomial Experiment**

1. The experiment consists of *n* identical trials.
2. Each trial results in one of two outcomes, success (S) or failure (F).
3. The probability of success on a single trial is *p* and remains constant from trial to trial. The probability of failure is $q = 1 - p$.
4. The trials are independent.
5. We are interested in *x*, the number of successes in *n* trials.

**The Binomial Probability Distribution**

- **For a binomial experiment with *n* trials and probability *p* of success on a given trial, the probability of *k* successes in *n* trials is**

$$P(x=k) = C_k^n p^k q^{n-k} = \frac{n!}{k!(n-k)!} p^k q^{n-k} \text{ for } k = 0,1,2,\ldots n.$$

$$\text{Recall} \quad C_k^n = \frac{n!}{k!(n-k)!}$$

$$\text{with } n! = n(n-1)(n-2)\ldots(2)1 \text{ and } 0! \equiv 1.$$

**The Mean and Standard Deviation**

- For a binomial experiment with *n* trials and probability *p* of success on a given trial, the measures of center and spread are:

$$\text{Mean} : \mu = np$$

$$\text{Variance} : \sigma^2 = npq$$

$$\text{Standard deviation} : \sigma = \sqrt{npq}$$

**Example**

A marksman hits a target 80% of the time. He fires five shots at the target. What is the probability that exactly 3 shots hit the target?

***n* = 5     *success* = hit     *p* = 0.8     *x* = # of hits**

$$P(x=3) = C_3^n p^3 q^{n-3} = \frac{5!}{3!2!}(.8)^3(.2)^{5-3}$$

$$= 10(.8)^3(.2)^2 = .2048$$

What is the probability that more than 3 shots hit the target?

$$P(x>3) = C_4^5 p^4 q^{5-4} + C_5^5 p^5 q^{5-5}$$

$$= \frac{5!}{4!1!}(.8)^4(.2)^1 + \frac{5!}{5!0!}(.8)^5(.2)^0$$

$$= 5(.8)^4(.2) + (.8)^5 = .7373$$

**Cumulative Probability Tables**

You can use the **cumulative probability tables** to find probabilities for selected binomial distributions.

- ✓ **Find the table for the correct value of *n*.**
- ✓ **Find the column for the correct value of *p*.**
- ✓ The row marked "$k$" gives the cumulative probability, $P(x \le k) = P(x = 0) + \ldots + P(x = k)$

## Example

| $k$ | $p$ = .80 |
|---|---|
| 0 | .000 |
| 1 | .007 |
| 2 | .058 |
| 3 | .263 |
| 4 | .672 |
| 5 | 1.000 |

$n = 5$

| | $p$ | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $k$ | .01 | .05 | .10 | .20 | .30 | .40 | .50 | .60 | .70 | .80 | .90 | .95 | .99 | $k$ |
| 0 | .951 | .774 | .590 | .328 | .168 | .078 | .031 | .010 | .002 | .000 | .000 | .000 | .000 | 0 |
| 1 | .999 | .977 | .919 | .737 | .528 | .337 | .188 | .087 | .031 | .007 | .000 | .000 | .000 | 1 |
| 2 | 1.000 | .999 | .991 | .942 | .837 | .683 | .500 | .317 | .163 | .058 | .009 | .001 | .000 | 2 |
| 3 | 1.000 | 1.000 | 1.000 | .993 | .969 | .913 | .812 | .663 | .472 | .263 | .081 | .023 | .001 | 3 |
| 4 | 1.000 | 1.000 | 1.000 | 1.000 | .998 | .990 | .969 | .922 | .832 | .672 | .410 | .226 | .049 | 4 |
| 5 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 5 |

**What is the probability that exactly 3 shots hit the target?**

**$P(x = 3)$** $= P(x \le 3) - P(x \le 2)$

$= .263 - .058$

$= .205$

What is the probability that more than 3 shots hit the target?

**$P(x > 3)$** $= 1 - P(x \le 3)$

$= 1 - .263 = .737$

**Example**

- Here is the probability distribution for ***x* = number of hits.** What are the mean and standard deviation for *x*?

$\text{Mean}: \mu = np = 5(.8) = 4$

$\text{Standard deviation}: \sigma = \sqrt{npq}$

$= \sqrt{5(.8)(.2)} = .89$

**Example**

- Would it be unusual to find that none of the shots hit the target?

$\mu = 4; \sigma = .89$

- The value $x = 0$ lies

$$z = \frac{x-\mu}{\sigma} = \frac{0-4}{.89} = -4.49$$

- more than 4 standard deviations below the mean. Very unusual.