Department of Statistics
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SATS 101

# Introduction to Probability and Statistics

## Yarmouk University

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## Chapter 4
## Probability and Probability Distributions

Basic Concepts:

- An **experiment** is the process by which an observation (or measurement) is obtained.
- **Experiment** **:** Record an age
  **:** Toss a die
  **:** Record an opinion (yes, no)
  **:** Toss two coins
- A **simple event** is the outcome that is observed on a single repetition of the experiment.
  - The basic element to which probability is applied.
  - One and only one simple event can occur when the experiment is performed.

- A **simple event** is denoted by E with a subscript.
- Each simple event will be assigned a probability, measuring "how often" it occurs.
- The set of all simple events of an experiment is called the **sample space, S.**

- An **event** is a collection of one or more **simple events.**
- Two events are **mutually exclusive** if, when one event occurs, the other cannot, and vice versa.

**The Probability of an Event**

- The probability of an event A measures "how often" we think A will occur. We write **P(A).**
- Suppose that an experiment is performed *n* times. The relative frequency for an event A is
- If we let *n* get infinitely large,

$$P(A)=\lim_{n\to\infty}\frac{f}{n}$$

- P(A) must be between 0 and 1.
  - If event A can never occur, P(A) = 0. If event A always occurs when the experiment is performed, P(A) =1.
- The sum of the probabilities for all simple events in S equals 1.
- The **probability of an event A** is found by adding the probabilities of all the simple events contained in A.

## Counting Rules

- If the simple events in an experiment are **equally likely,** you can calculate

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

- You can use **counting rules** to find $n_A$ and $N$.

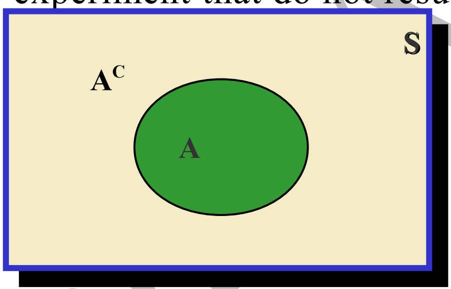
**Example:**
Toss three coins. The total number of simple events is:
**$2 \times 2 \times 2 = 8$**

**Example:**
Toss two dice. The total number of simple events is: **$6 \times 6 = 36$**

**Example:**
Two M&Ms are drawn from a dish containing two red and two blue candies. The total number of simple events is: **$4 \times 3 = 12$**

## Event Relations

- The **complement** of an event **A** consists of all outcomes of the experiment that do not result in event A. We write **$A^C$.**




## Example

- Select a student from the classroom and record his/her **hair color** and **gender.**

Mutually exclusive; B = $C^C$

- **A:** student has brown hair
- **B:** student is female
- **C:** student is male

- What is the relationship between events **B** and **C**?
- **$A^C$: Student does not have brown hair**
- **$B \cap C$: Student is both male and female = $\varnothing$**
- **$B \cup C$:** Student is either male and female = all students = S

**Calculating Probabilities for Unions and Complements**

- There are special rules that will allow you to calculate probabilities for composite events.
- <u>**The Additive Rule for Unions:**</u>
- For any two events, **A** and **B**, the probability of their union, **P(A∪B)**, is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Example: Additive Rule**

Suppose that there were 120 students in the classroom, and that they could be classified as follows:

| | Brown | Not Brown |
|---|---|---|
| **Male** | **20** | **40** |
| **Female** | **30** | **30** |

**A:** brown hair, **P(A) = 50/120**
**B:** female, **P(B) = 60/120**
Check: P(A∪B)
= (20 + 30 + 30)/120

P(A∪B) = P(A) + P(B) – P(A∩B)
= 50/120 + 60/120 - 30/120
= 80/120 = 2/3

**A Special Case**

When two events A and B are **mutually exclusive,** P(A∩B) = 0 and **P(A∪B) = P(A) + P(B).**

| | Brown | Not Brown |
|---|---|---|
| **Male** | **20** | **40** |
| **Female** | **30** | **30** |

**A:** male with brown hair
P(A) = 20/120
**B:** female with brown hair
P(B) = 30/120
A and B are mutually exclusive, so that
P(A∪B) = P(A) + P(B)
= 20/120 + 30/120
= 50/120

**Calculating Probabilities for Complements**

- We know that for any event **A:**
  - $\mathbf{P(A \cap A^C) = 0}$
- Since either **A** or **AC** must occur,

$\mathbf{P(A \cup A^C) = 1}$

- so that $\mathbf{P(A \cup A^C) = P(A) + P(A^C) = 1}$

$\mathbf{P(A^C) = 1 - P(A)}$

**Example**

Select a student at random from the classroom. Define:

| | **Brown** | **Not Brown** |
|---|---|---|
| **Male** | **20** | **40** |
| **Female** | **30** | **30** |

**A:** male

$P(A) = 60/120$

**B:** female

A and B are complementary, so that

$P(B) = 1 - P(A)$

$= 1 - 60/120 = 60/120$

**Calculating Probabilities for Intersections**

- In the previous example, we found P(A ∩ B) directly from the table. Sometimes this is impractical or impossible. The rule for calculating P(A ∩ B) depends on the idea of **independent and dependent events.**

Two events, **A** and **B**, are said to be **independent** if and only if the probability that event **A** occurs does not change, depending on whether or not event **B** has occurred.

**Example 1**

- Toss a fair coin twice. Define
  - A: head on second toss
  - B: head on first toss

P(A|B) = ½
P(A|not B) = ½
P(A) does not change, whether B happens or not…
A and B are independent!

**Example 2**

- A bowl contains five M&Ms®, two red and three blue. Randomly select two candies, and define
  - A: second candy is red.
  - B: first candy is blue.

P(A|B) =P(2nd red|1st blue)= 2/4 = 1/2
P(A|not B) = P(2nd red|1st red) = 1/4

P(A) does change, depending on whether B happens or not…
A and B are dependent!

**Defining Independence**

- We can redefine independence in terms of conditional probabilities:

Two events A and B are independent if and only if
P(A|B) = P(A) or P(B|A) = P(B)
Otherwise, they are dependent.

- Once you've decided whether or not two events are independent, you can use the following rule to calculate their intersection.

**The Multiplicative Rule for Intersections**

- For any two events, **A** and **B**, the probability that both **A** and **B** occur is

**P(A∩B) = P(A) P(B given that A occurred) = P(A)P(B|A)**

- If the events **A** and **B** are independent, then the probability that both **A** and **B** occur is

**P(A∩B) = P(A) P(B)**

**Example 1**

In a certain population, 10% of the people can be classified as being high risk for a heart attack. Three people are randomly selected from this population. What is the probability that exactly one of the three are high risk?

Define H: high risk N: not high risk

P(exactly one high risk) = P(HNN) + P(NHN) + P(NNH)

= P(H)P(N)P(N) + P(N)P(H)P(N) + P(N)P(N)P(H)

= (.1)(.9)(.9) + (.9)(.1)(.9) + (.9)(.9)(.1)= $3(.1)(.9)^2$ = .243

**Example 2**

Suppose we have additional information in the previous example. We know that only 49% of the population are female. Also, of the female patients, 8% are high risk. A single person is selected at random. What is the probability that it is a high risk female?

Define H: high risk F: female

From the example, P(F) = .49 and P(H|F) = .08. Use the Multiplicative Rule:

P(high risk female) = P(H∩F)

= P(F)P(H|F) =.49(.08) = .0392

**The Mean and Standard Deviation**

- Let *x* be a discrete random variable with probability distribution *p(x)*. Then the mean, variance and standard deviation of *x* are given as

Mean $: \mu = \sum x p(x)$

Variance $: \sigma^2 = \sum (x-\mu)^2 p(x)$

Standard deviation $: \sigma = \sqrt{\sigma^2}$

**Example**

- Toss a fair coin 3 times and record *x* the number of heads.

| *x* | *p(x)* | *xp(x)* | *(x-μ)2p(x)* |
|---|---|---|---|
| 0 | 1/8 | 0 | (-1.5)2(1/8) |
| 1 | 3/8 | 3/8 | (-0.5)2(3/8) |
| 2 | 3/8 | 6/8 | (0.5)2(3/8) |
| 3 | 1/8 | 3/8 | (1.5)2(1/8) |

$$\mu = \Sigma xp(x) = \frac{12}{8} = 1.5$$

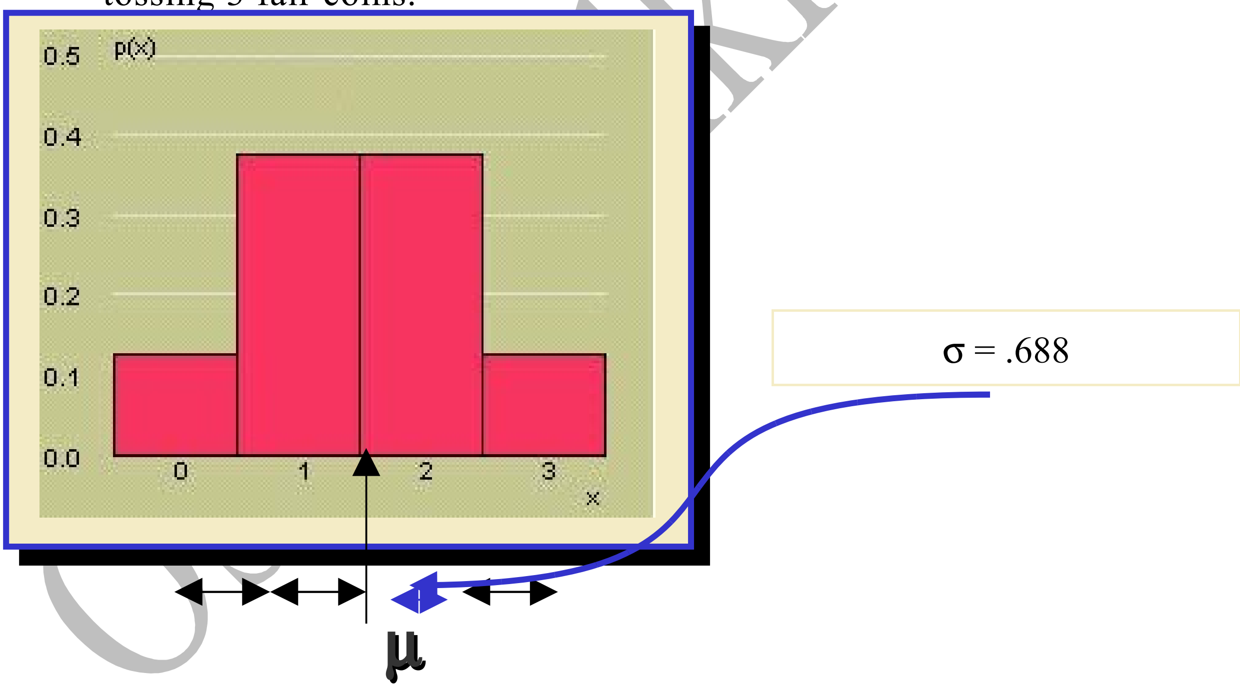
$$\sigma^2 = \Sigma(x-\mu)^2\, p(x)$$

$$\sigma^2 = .28125 \;\; + .09375 \;\; + .09375 \;\; + .28125 \;\; = .75$$

$$\sigma = \sqrt{.75} = .688$$

**Example**

- The probability distribution for *x* the number of heads in tossing 3 fair coins.




- Shape? Symmetric; mound-shaped
- Outliers? None
- Center? m = 1.5
- Spread? s = .688

**Key Concepts**

**I. Experiments and the Sample Space**

1. Experiments, events, mutually exclusive events, simple events
2. The sample space
3. Venn diagrams, tree diagrams, probability tables

**II. Probabilities**

1. Relative frequency definition of probability
2. Properties of probabilities
   a. Each probability lies between 0 and 1.
   b. Sum of all simple-event probabilities equals 1.
3. P(A), the sum of the probabilities for all simple events in A

**III. Counting Rules**

1. *mn* Rule; extended *mn* Rule
2. Permutations: $P_r^n = \dfrac{n!}{(n-r)!}$
3. Combinations: $C_r^n = \dfrac{n!}{r!(n-r)!}$

**IV. Event Relations**

1. Unions and intersections
2. Events
   a. Disjoint or mutually exclusive: $P(A \cap B) = 0$
   b. Complementary: $P(A) = 1 - P(AC)$
3. Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$
4. Independent and dependent events
5. Additive Rule of Probability:
   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
6. Multiplicative Rule of Probability: $P(A \cap B) = P(A)P(B \mid A)$
7. Law of Total Probability
8. Bayes' Rule

**V. Discrete Random Variables and Probability Distributions**

1. Random variables, discrete and continuous
2. Properties of probability distributions
   $0 \le p(x) \le 1$ and $\sum p(x) = 1$
3. Mean or expected value of a discrete random variable:
   Mean : $\mu = \sum xp(x)$
4. Variance and standard deviation of a discrete random variable:
   Variance : $\sigma^2 = \sum (x-\mu)^2 p(x)$
   Standard deviation : $\sigma = \sqrt{\sigma^2}$